Precalculus

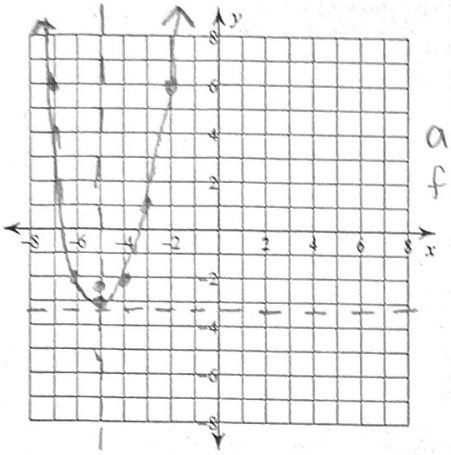
Name Key

# Conics Study Guide

Date

**Graph the following conic sections. Find all applicable information.**

1) $y=(x+5)^2-3$ Vertex $(-5,-3)$



$1=\frac{1}{4p}$

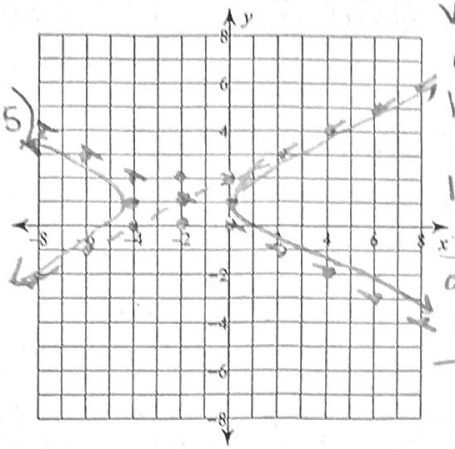
$p=\frac{1}{4}$

a.o.s. $x=-5$

focus $(-5,-2.75)$

directrix $y=-3.25$

2) $\frac{(x+2)^2}{4}-(y-1)^2=1$ Hyperbola



center $(-2,1)$

vertices $(0,1)$ & $(-4,1)$

length of Transverse axis $=4$

length of conjugate axis $=2$

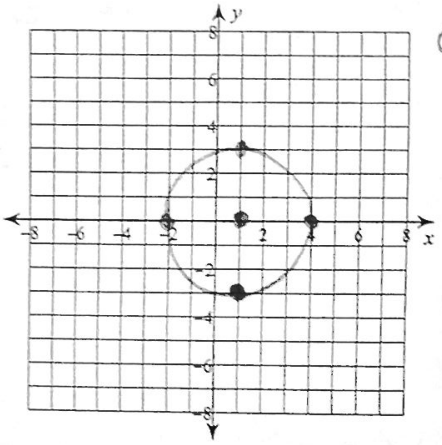
asymptotes $y-1=\pm\frac{1}{2}(x-1)$

$1^2+2^2=c^2$

$\sqrt{5}=c$

foci $(-2\pm\sqrt{5},1)$

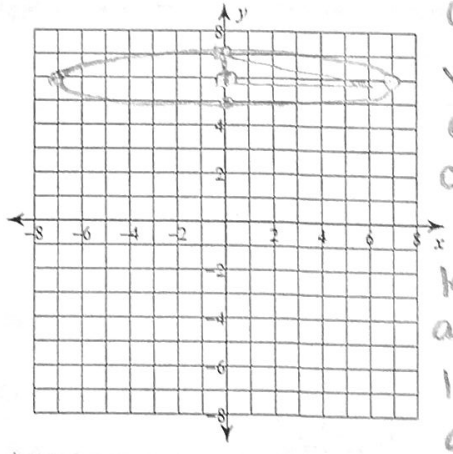
3) $(x-1)^2+y^2=9$ Circle



center $(1,0)$

radius $=3$

4) $\frac{x^2}{49}+(y-6)^2=1$ Ellipse



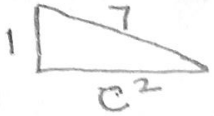
center $(0,6)$

vertices $(7,6)$ & $(-7,6)$

co-vertices $(0,7)$ & $(0,5)$

length of major axis $=14$

length of minor axis $=2$



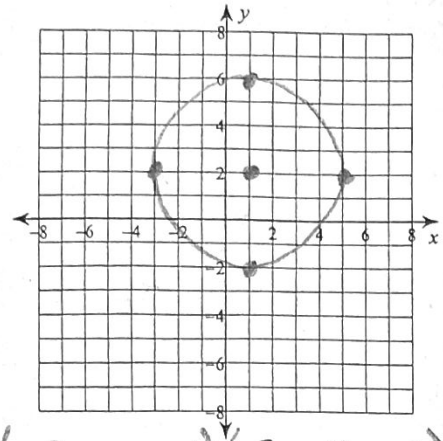

$1^2+c^2=7^2$

$c^2=48$

$c=\pm4\sqrt{3}$

foci $(-4\sqrt{3},6)$ $(4\sqrt{3},6)$

5) $x^2 + y^2 - 2x - 4y - 11 = 0$



$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 11 + 1 + 4$

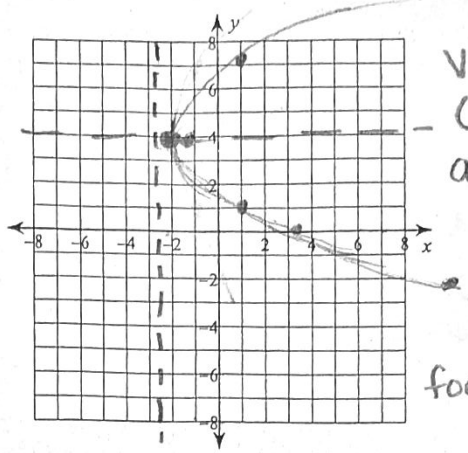
$(x-1)^2 + (y-2)^2 = 16$

Circle

center $(1,2)$

radius $= 4$

6) $-y^2 + 3x + 8y - 10 = 0$



vertex $(-2,4)$

a.o.s. directrix

$y = 4$ $x = -2\frac{3}{4}$

$\frac{1}{4p} = \frac{1}{3}$ $4p = 3$ $p = \frac{3}{4}$

focus $(-1\frac{1}{4}, 4)$

| x | y |
|---|---|
| 10 | -2 |
| 1 | 1 |

$3x = y^2 - 8y + 10$

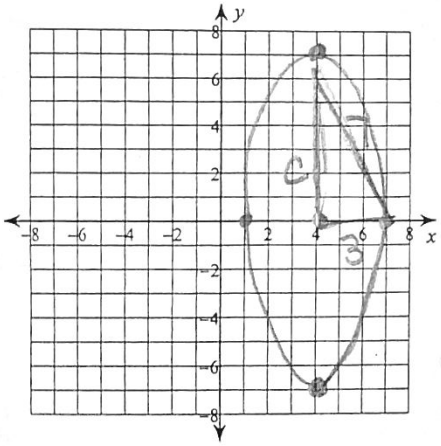
$3x = (y^2 - 8y + 16) + 10 - 16$

$\frac{3x}{3} = \frac{(y-4)^2}{3} - \frac{6}{3}$

$x = \frac{1}{3}(y-4)^2 - 2$

horizontal hyperbola

7) $49x^2 + 9y^2 - 392x + 343 = 0$



$c^2 + 3^2 = 7^2$

$c^2 = 40$

$c = \pm 2\sqrt{10}$

foci

$(3, 2\sqrt{10})(3, -2\sqrt{10})$

$49x^2 - 392x + 9y^2 = -343$

$49(x^2 - 8x + 16) + 9y^2 = -343 + 784$

$\frac{49(x-4)^2}{441} + \frac{9y^2}{441} = \frac{441}{441}$

$\frac{(x-4)^2}{9} + \frac{y^2}{49} = 1$

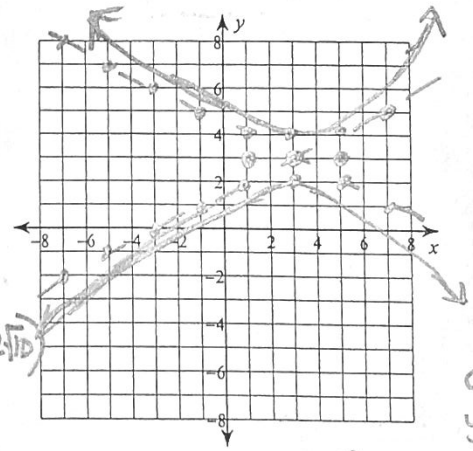
Vertical Ellipse

center $(4,0)$

vertices $(4,-7)$ & $(4,7)$

co-vertices $(1,0)$ & $(7,0)$

8) $-x^2 + 4y^2 + 6x - 24y + 23 = 0$



$1^2 + 2^2 = c^2$

$\sqrt{5} = c$

foci

$(3, 3 \pm \sqrt{5})$

asymptotes

$y - 3 = \pm\frac{1}{2}(x - 3$

$= -23$

$4y^2 - 24y - x^2 + 6x$

$4(y^2 - 6y + 9) - (x^2 - 6x + 9) = -23 + 36 - 9$

$\frac{4(y-3)^2}{4} - \frac{(x-3)^2}{4} = \frac{4}{4}$

$\frac{(y-3)^2}{1} - \frac{(x-3)^2}{4} = 1$

vertical hyperbola

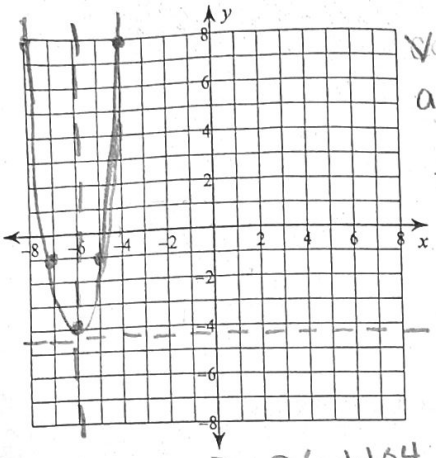
center $(3,3)$

vertices $(3,2)(3,4)$

length of transverse axis $= 4$

length of conjugate axis $= 2$

9) $-3x^2 - 36x + y - 104 = 0$



vertex $(-6,-4)$

a.o.s. $x=-6$

$\frac{1}{4p} = \frac{3}{1}$

$12p = 1$

$p = \frac{1}{12}$

focus $= (-6, -3\frac{11}{12})$

directrix

$y = -4\frac{1}{12}$

$y = 3x^2 + 36 + 104$

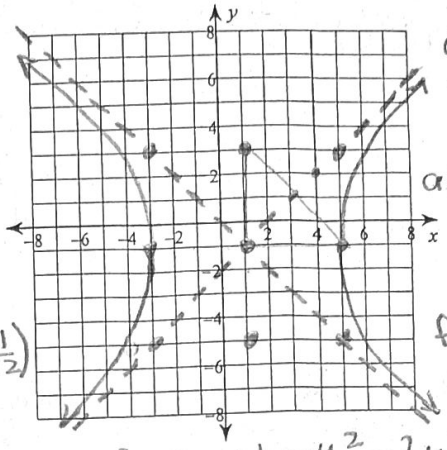
$y = (3x^2 + 36x\ \ ) + 104$

$y = 3(x^2 + 12x + 36) + 104 - 108$

$y = 3(x+6)^2 - 4$

Vertical parabola

10) $x^2 - y^2 - 2x - 2y - 16 = 0$



center $(1,-1)$

vertices

$(-3,-1)\ (5,-1)$

asymptotes

$y+1 = \pm 1(x-1)$

$c = \pm 4\sqrt{2}$

foci

$(1 \pm 4\sqrt{2}, -1)$

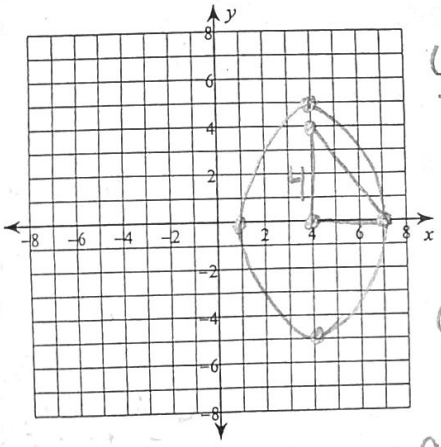
$x^2 - 2x + 1 - y^2 - 2y = 16 + 1$

$(x^2 - 2x + 1) - (y^2 + 2y + 1) = 16 + 1 - 1$

$\frac{(x-1)^2}{16} - \frac{(y+1)^2}{16} = \frac{16}{16}$

$\frac{(x-1)^2}{16} - \frac{(y+1)^2}{16} = 1$ Horiz. Hyperbola

11) $25x^2 + 9y^2 - 200x + 175 = 0$



$\frac{(x-4)^2}{9} + \frac{y^2}{25} = 1$

Vertical ellipse

center $(4,0)$

vertices $(4,5)$ & $(4,-5)$

Co-vertices $(1,0)\ (7,0)$

length of major axis $= 10$

length of minor axis $= 6$

$c^2 = 4^2 = 5^2$

$c^2 = 16$

foci $(4,4)\ (4,-4)$

$25x^2 - 200x + 9y^2 = -175$

$25(x^2 - 8x + 16) + 9y^2 = -175 + 400$

$\frac{25(x-4)^2}{225} + \frac{9y^2}{225} = \frac{225}{225}$

**Use the information provided to write the standard form equation of each circle.**

12) Center: $(10, 16)$
Point on Circle: $(11, 15)$

$r = \sqrt{(11-10)^2 + (15-16)^2}$

$r = \sqrt{1+1} = \sqrt{2}$

$\boxed{(x-10)^2 + (y-16)^2 = 2}$

13) Center: $(-10, -10)$
Radius: 5

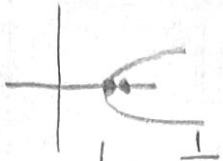
$(x+10)^2 + (y+10)^2 = 25$

**Use the information provided to write the vertex form equation of each parabola.**

14) Vertex: $(8, 0)$, Focus: $\left(8, \frac{1}{12}\right)$ $p = \frac{1}{12}$

$x = 3(y-0)^2 + 8$

$\boxed{x = 3y^2 + 8}$

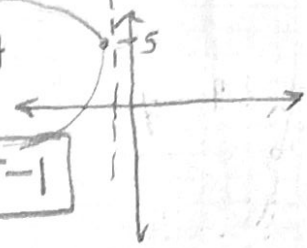


$a = \frac{1}{4\left(\frac{1}{12}\right)} = \frac{1}{\frac{1}{3}} = 3$

15) Vertex: $(-1, 5)$, Directrix: $x = -\frac{15}{16}$

$p = -\frac{1}{16}$

$a = \frac{1}{4p} = \frac{1}{-\frac{1}{4}} = -4$

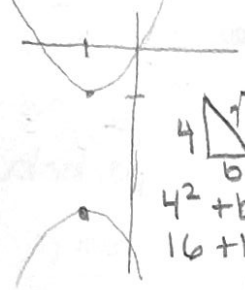



$\boxed{x = -4(y-5)^2 - 1}$

**Use the information provided to write the standard form equation of each hyperbola.**

16) Vertices: $(-3, -3)$, $(-3, -11)$
Foci: $(-3, -7 + \sqrt{137})$, $(-3, -7 - \sqrt{137})$

center $(-3, -7)$ $c = \pm\sqrt{137}$

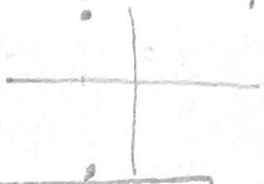


$4 \quad \sqrt{137}$
$b$

$4^2 + b^2 = (\sqrt{137})^2$
$16 + b^2 = 137$
$b^2 = 121$
$b = 11$

$\boxed{\frac{(y+7)^2}{16} - \frac{(x+3)^2}{121} = 1}$

17) Vertices: $(-3, 7)$, $(-3, -11)$ center $(-3, -2)$
Asymptotes: $y = \frac{9}{7}x + \frac{13}{7}$
$y = -\frac{9}{7}x - \frac{41}{7}$



$\boxed{\frac{(y+2)^2}{81} - \frac{(x+2)^2}{49} = 1}$

**Use the information provided to write the standard form equation of each ellipse.**

18) Vertices: $(5, 8)$, $(-13, 8)$ center $(-4, 8)$
Foci: $(-4 + \sqrt{65}, 8)$, $(-4 - \sqrt{65}, 8)$ $c = \sqrt{65}$

$\frac{(x+4)^2}{9^2} + \frac{(y-8)^2}{4^2} = 1$

$b \quad 9$
$\sqrt{65}$

$b^2 + (\sqrt{65})^2 = 9^2$
$b^2 = 16$
$b = 4$

$\boxed{\frac{(x+4)^2}{81} + \frac{(y-8)^2}{16} = 1}$

19) Center: $(7, 3)$
Vertex: $(0, 3)$
Focus: $(7 + 3\sqrt{5}, 3)$ $c = 3\sqrt{5}$

$b^2 + (3\sqrt{5})^2 = 7^2$
$b^2 + 45 = 49$
$b^2 = 4$
$b = 2$

$\frac{(x-7)^2}{(3\sqrt{5})^2} + \frac{(y-3)^2}{2^2} = 1$

$\boxed{\frac{(x-7)^2}{49} + \frac{(y-3)^2}{4} = 1}$