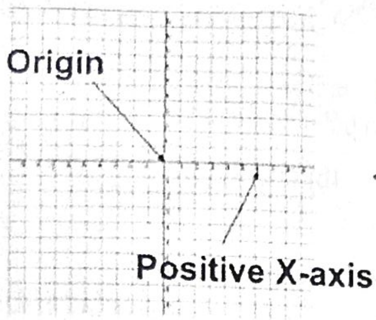
Pre-Calculus

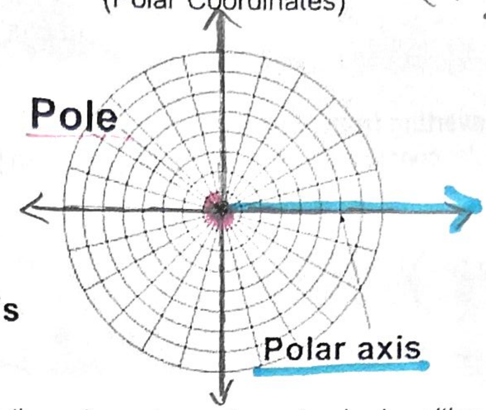
Name: Key

Notes—(8.1) Polar Coordinates/Equations

(x, y) Rectangular Grid (Cartesian Coordinates)

Polar Grid (Polar Coordinates) (r, θ)





A **polar coordinate system** is a plane with a point O, the **pole**, and a ray from *standard position,* 0, the **polar axis**. Each point P in the plane is assigned as **polar coordinates** as follows: r is the **directed distance** from O to P and θ is the **directed angle** whose initial side is on the polar axis and whose terminal side is on the line OP.

As in trigonometry, we measure θ as positive when moving counterclockwise and negative when moving clockwise. If $r > 0$, then P is on the terminal side of θ. If $r < 0$, then P is on the terminal side of $\theta + \pi$. We can use radian or degree measure for the angle θ.

EXAMPLE 1 Plotting Points in the Polar Coordinate System

Plot the points with the given polar coordinates.

(a) $P(2, \pi/3)$

$\left(2, -\frac{5\pi}{3}\right)$

$\left(-2, -\frac{2\pi}{3}\right)$

$\left(-2, \frac{4\pi}{3}\right)$

(b) $Q(-1, 3\pi/4)$

$\left(1, \frac{7\pi}{4}\right)$ $\left(1, -\frac{\pi}{4}\right)$

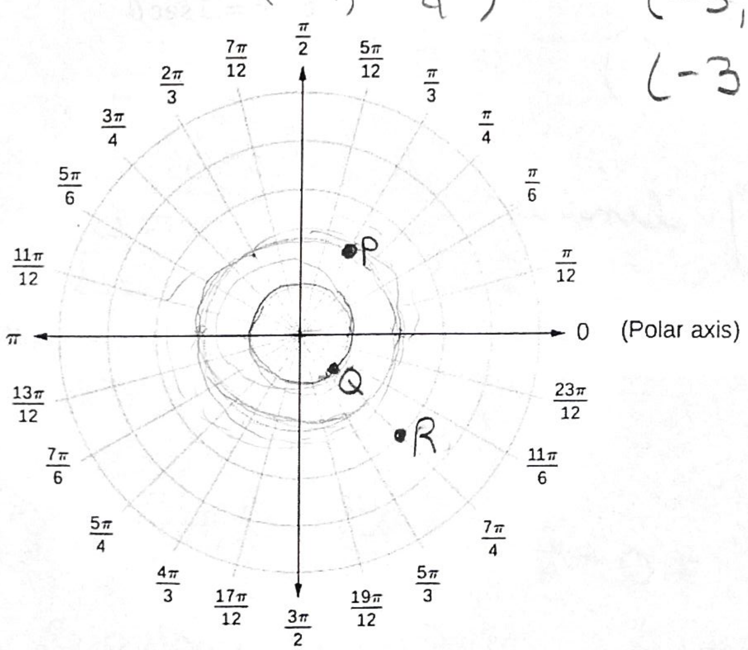
$\left(-1, -\frac{5\pi}{4}\right)$

(c) $R(3, -45°)$

$(3, 315°)$

$(-3, 135°)$

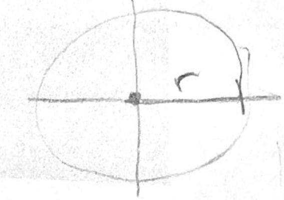
$(-3, -225°)$



Coordinate Conversion Equations

Let the point P have polar coordinates (r, θ) and rectangular coordinates (x, y). Then

$$x = r\cos\theta, \qquad r^2 = x^2 + y^2,$$
$$y = r\sin\theta, \qquad \tan\theta = \frac{y}{x}.$$



EXAMPLE 2 Converting from Polar to Rectangular Coordinates
Find the rectangular coordinates of the points with the given polar coordinates.

(a) $P(3, 5\pi/6)$ (x, y)

$\left(3\cos\frac{5\pi}{6}, 3\sin\frac{5\pi}{6}\right)$ $(r\cos\theta, r\sin\theta)$

$\left(3\left(\frac{\sqrt{3}}{2}\right), 3\left(\frac{1}{2}\right)\right)$

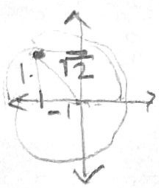
$\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

(b) $Q(2, -200°)$

$(2\cos(-200°), 2\sin(-200°))$

$\approx(-1.879, .684)$

EXAMPLE 3 Converting from Rectangular to Polar Coordinates
Find two polar coordinate pairs for the points with given rectangular coordinates.

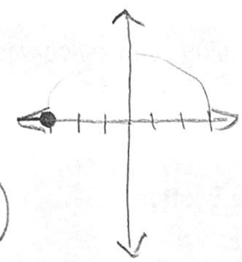


(a) $P(-1, 1)$

$\left(\sqrt{2}, \frac{3\pi}{4}\right)$

$\left(-\sqrt{2}, -\frac{\pi}{4}\right)$

$\left(-\sqrt{2}, \frac{7\pi}{4}\right)$



(b) $Q(-3, 0)$

$(3, \pi)$

$(-3, 0)$

$(3, -\pi)$

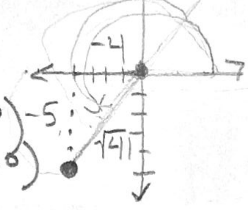
(c) $(-4, -5)$

$(\sqrt{41}, 231.34°)$

$(-\sqrt{41}, 51.34°)$

$\tan\theta = \frac{5}{4}$

$\theta = \tan^{-1}\left(\frac{5}{4}\right)$



Converting from Polar Form to Rectangular Form

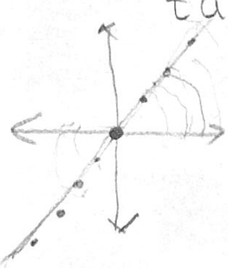
EXAMPLE 4 Convert each of the following to rectangular form and then graph.

a) $\theta = \frac{\pi}{4}$

$\tan\theta = \tan\left(\frac{\pi}{4}\right)$

$\frac{y}{x} = 1$

$\boxed{y = x}$ linear



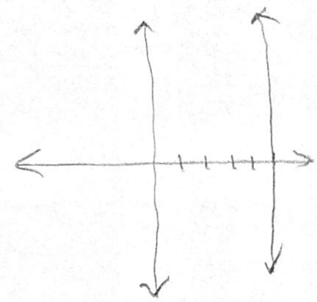
b) $r = 5\sec\theta$

$r = \frac{5}{\cos\theta}$

$r\cos\theta = 5$

$\boxed{x = 5}$

vertical line



c) $r = \cos\theta$

$r^2 = r\cos\theta$

$x^2 + y^2 = x$

$x^2 - x + \frac{1}{4} + y^2 = 0 + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$

circle

center $\left(\frac{1}{2}, 0\right)$ $r = \frac{1}{2}$

d) $r^2 = 3^2$

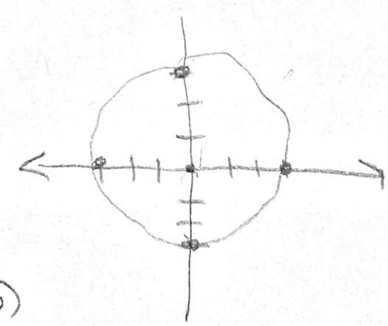
$r^2 = 9$

$x^2 + y^2 = 9$

circle

center $(0, 0)$

$r = 3$



EXAMPLE 5 Convert each of the following to rectangular form and identify the conic..

a) $r = \frac{4}{3-2\cos\theta}$

$3r - 2r\cos\theta = 4$

$3r - 2x = 4$

$(3r)^2 = (2x+4)^2$

$9r^2 = 4x^2 + 16x + 16$

$9x^2 + 9y^2 = 4x^2 + 16x + 16$

$5x^2 + 9y^2 - 16x - 16 = 0$

Ellipse

b) $r = \frac{1}{1+\sin\theta}$

$r + r\sin\theta = 1$

$r + y = 1$

$r^2 = (1-y)^2$

$x^2 + \cancel{y^2} = 1 - 2y + \cancel{y^2}$

$\frac{x^2 - 1}{-2} = \frac{-2y}{-2}$

$\boxed{y = -\frac{1}{2}x^2 + \frac{1}{2}}$

Parabola

c) $r = \sin\theta - \cos\theta$

$r^2 = r\sin\theta - r\cos\theta$

$x^2 + y^2 = y - x$

$x^2 + y^2 + x - y = 0$

$\boxed{\text{Circle}}$

EXAMPLE 6 Convert each of the following to polar form.

a) $\boxed{x^2 + y^2} = 5$

$r^2 = 5$

$\boxed{r = \sqrt{5}}$

b) $(x-2)^2 + y^2 = 4$

$x^2 - 4x + \cancel{4} + y^2 = \cancel{4}$

$r^2 - 4r\cos\theta = 0$

$r(r - 4\cos\theta) = 0$

$\cancel{r=0}$ $\quad r - 4\cos\theta = 0$

$\boxed{r = 4\cos\theta}$

c) $(x+4)^2 + (y-1)^2 = 17$

$x^2 + 8x + \cancel{16} + y^2 - 2y + \cancel{1} = \cancel{17}$

$r^2 + 8r\cos\theta - 2r\sin\theta = 0$

$\cancel{r=0}$ $\quad r + 8\cos\theta - 2\sin\theta = 0$

$\boxed{r = -8\cos\theta + 2\sin\theta}$