Converting Polar/Rectangular Equations

Convert the rectangular equation into polar form.

1) $x^2 + y^2 - 121 = 0$

$$r^2 = 121$$

$$\boxed{r = 11}$$

✗ $x^2 + y^2 - 49 = 0$

$$r^2 = 49$$

$$\boxed{r = 7}$$

✗ $x^2 + y^2 - 28x + 22y = 0$

$$r^2 - 28r\cos\theta + 22r\sin\theta = 0$$

$$\boxed{r = 28\cos\theta - 22\sin\theta}$$

✗ $x^2 + y^2 - 22x + 30y = 0$

$$r^2 - 22r\cos\theta + 30r\sin\theta = 0$$

$$\boxed{r = 22\cos\theta - 30\sin\theta}$$

5) $x^2 + y^2 - 22x + 10y = 0$

$$r^2 - 22r\cos\theta + 10r\sin\theta = 0$$

$$\boxed{r = 22\cos\theta - 10\sin\theta}$$

6) $(x-3)^2 + (y-4)^2 = 25$

$$x^2 - 6x + 9 + y^2 - 8y + 16 = 25$$
$$x^2 + y^2 - 6x - 8y = 0$$
$$r^2 - 6r\cos\theta - 8r\sin\theta = 0$$

$$\boxed{r = 6\cos\theta + 8\sin\theta}$$

7) $(x+2)^2 + (y+6)^2 = 40$

$$x^2 + 4x + 4 + y^2 + 12y + 36 = 40$$
$$x^2 + y^2 + 4x + 12y = 0$$
$$r^2 + 4r\cos\theta + 12r\sin\theta = 0$$

$$\boxed{r = -4\cos\theta - 12\sin\theta}$$

8) $(x-11)^2 + (y-2)^2 = 125$

$$x^2 - 22x + 121 + y^2 - 4y + 4 = 125$$
$$x^2 + y^2 - 22x - 4y = 0$$
$$r^2 - 22r\cos\theta - 4r\sin\theta = 0$$

$$\boxed{r = 22\cos\theta + 4\sin\theta}$$

9) $x = 16$

$r\cos\theta = 16$

$r = \dfrac{16}{\cos\theta}$

$\boxed{r = 16\sec\theta}$

10) $y = x$

$\dfrac{y}{x} = 1$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\boxed{\theta = \dfrac{\pi}{4}}$

12) $4x + 5y = -40$

$4r\cos\theta + 5r\sin\theta = -40$

$r(4\cos\theta + 5\sin\theta) = -40$

$\boxed{r = \dfrac{-40}{4\cos\theta + 5\sin\theta}}$

11) $y = \dfrac{\sqrt{3}}{3}x$

$\boxed{\theta = \dfrac{\pi}{6}}$

$\dfrac{y}{x} = \dfrac{\sqrt{3}}{3}$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

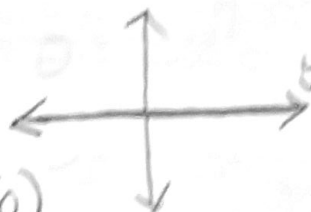$\theta = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

13) $y = 0$

$r\sin\theta = 0$

$r \neq 0 \quad \sin\theta = 0$

$\theta = \sin^{-1}(0)$

$\boxed{\theta = 0 \quad \text{or} \quad \theta = \pi}$

14) $y = 6x + 5$

$r\sin\theta = 6r\cos\theta + 5$

$r\sin\theta - 6r\cos\theta = 5$

$r(\sin\theta - 6\cos\theta) = 5$

$\boxed{r = \dfrac{5}{\sin\theta - 6\cos\theta}}$

Convert the polar equation into rectangular form.

15) $\theta = \dfrac{5\pi}{6}$

$\tan\theta = \tan\left(\dfrac{5\pi}{6}\right)$

$\dfrac{y}{x} = \dfrac{\sqrt{3}}{3}$

$\boxed{y = \dfrac{\sqrt{3}}{3}x}$

line
slope $= \sqrt{3}/3$
y-int $(0,0)$

16) $\theta = -\dfrac{\pi}{6}$

$\tan\theta = \tan\left(\dfrac{-\pi}{6}\right)$

$\dfrac{y}{x} = -\dfrac{\sqrt{3}}{3}$

$\boxed{y = -\dfrac{\sqrt{3}}{3}x}$

line slope $= \dfrac{-\sqrt{3}}{3}$
y-int $(0,0)$

17) $r^2 = 4^2$

$\boxed{x^2 + y^2 = 16}$

circle
center $(0,0)$
radius $= 4$

18) $r = 2^2$

$r^2 = 4$

$\boxed{x^2 + y^2 = 4}$

circle
center $(0,0)$ $r = 2$

19) $r = 4\sin\theta$

$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$

$x^2 + y^2 - 4y + 4 = 0 + 4$

$\boxed{x^2 + (y-2)^2 = 4}$
circle, center $(0,2)$ radius $= 2$

20) $r = -2\sin\theta$

$r^2 = -2r\sin\theta$

$x^2 + y^2 = -2y$

$x^2 + y^2 - 2y + 1 = 0 + 1$

$\boxed{x^2 + (y-1)^2 = 1}$ Circle center $(0,1)$
$r = 1$

21) $r = \dfrac{3}{\cos \theta}$

$r\cos\theta = 3$

$\boxed{x = 3}$

Vertical line

22) $r = 8^2$

$\boxed{x^2 + y^2 = 64}$

center $(0,0)$

$r = 8$

23) $r = 5\cos\theta - 8\sin\theta$

$r^2 = 5r\cos\theta - 8r\sin\theta$

$x^2 + y^2 = 5x - 8y$

$x^2 - 5x + \frac{25}{4} + y^2 + 8y + 16 = \frac{25}{4} + 16$

$\boxed{\left(x - \frac{5}{2}\right)^2 + (y+4)^2 = \frac{89}{4}}$

center $\left(\frac{5}{2}, -4\right)$. $r = \frac{\sqrt{89}}{2}$

24) $r = -8\cos\theta + 10\sin\theta$

$r^2 = -8r\cos\theta + 10r\sin\theta$

$x^2 + y^2 = -8x + 10y$

$x^2 + 8x + 16 + y^2 - 10y + 25 = 16 + 25$

$\boxed{(x+4)^2 + (y-5)^2 = 41}$

center $(-4, 5)$, $r = \sqrt{41}$

25) $r = -\dfrac{11}{\sin\theta}$

$r\sin\theta = -11$

$\boxed{y = -11}$ horizontal line

26) $r = 4\csc\theta$

$r = \dfrac{4}{\sin\theta}$

$r\sin\theta = 4$

$\boxed{y = 4}$ horizontal line

27) $r = 9\sec\theta$

$r = \dfrac{9}{\cos\theta}$

$r\cos\theta = 9$

$\boxed{x = 9}$

$\boxed{\text{Vertical line}}$

28) $\theta = 315°$

$\tan\theta = \tan(315°)$

$\dfrac{y}{x} = -1$

$\boxed{y = -x}$ line slope $= -1$ y int $(0,0)$

29) $r = \dfrac{4}{1 - \cos\theta}$

$r - r\cos\theta = 4$

$r - x = 4$

$r = x + 4$

$r^2 = (x+4)^2$

$x^2 + y^2 = x^2 + 8x + 16$

$\dfrac{y^2}{8} - \dfrac{16}{8} = \dfrac{8x}{8}$

$\boxed{x = \frac{1}{8}y^2 - 2}$ parabola Vertex $(-2, 0)$

30) $r = \dfrac{3}{3 + \sin\theta}$

$3r + r\sin\theta = 3$

$3r + y = 3$

$3r = 3 - y$

$(3r)^2 = (3 - y)^2$

$9r^2 = 9 - 6y + y^2$

$9x^2 + 9y^2 = 9 - 6y + y^2$

$\boxed{9x^2 + 8y^2 + 6y - 9 = 0}$

Ellipse

31) $r = \dfrac{2}{3+2\cos\theta}$

$3r + 2r\cos\theta = 2$

$3r + 2x = 2$

$(3r)^2 = (2-2x)^2$

$9r^2 = 4 - 8x + 4x^2$

$9x^2 + 9y^2 = 4 - 8x + 4x^2$

$\boxed{5x^2 + 9y^2 + 8x - 4 = 0}$

Ellipse

32) $r = \dfrac{5}{2-4\sin\theta}$

$2r - 4r\sin\theta = 5$

$2r - 4y = 5$

$(2r)^2 = (4y+5)^2$

$4r^2 = 16y^2 + 40y + 25$

$4x^2 + 4y^2 = 16y^2 + 40y + 25$

$\boxed{4x^2 - 12y^2 - 40y - 25 = 0}$

Hyperbola