Pre-Calculus Name: ______________________________

Notes—(8.4) Parametric Equations & Motion

DEFINITIONS: The graph of the ordered pairs (x, y) where: $x = f(t),\ y = g(t)$ are functions defined on an interval I of t-values is a **PARAMETRIC CURVE**. The equations are **PARAMETRIC EQUATIONS** for the curve, the variable t is a **PARAMETER**, and I is the **PARAMETER INTERVAL**.

*When we give parametric equations and a parameter interval for a curve, we have "**parametrized"** the curve.

A "parametrization"** of a curve consists of the *parametric equations* AND the *interval* of *t*-values.

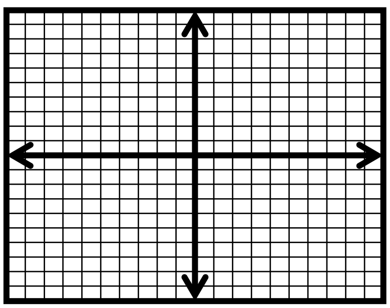
EX 1) Given the parametric equation, and value for the parameter, find the coordinate of the point on the plane curve.

a) $x(t) = 3 - 5t, y(t) = 4 + 2t; t = 1$

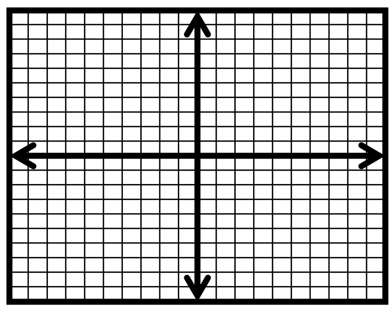
b) $x(t) = 4 + 2\cos t; y(t) = 3 + 5\sin t; t = \frac{\pi}{2}$

Ex 2) Graphing Parametric Equations: For the given parameter interval, graph the parametric equations using your graphing calculator & note the difference you see. $x = t^2 - 2,\ y = 3t$

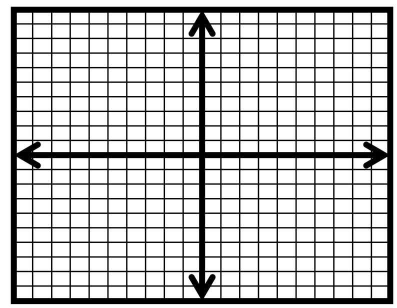
(a) $-3 \le t \le 1$



(b) $-2 \le t \le 3$



(c) $-3 \le t \le 3$



*What do you think is the point of doing this example?

If we do not specify a parameter interval for the parametric equations $x = f(t),\ y = g(t)$, it is understood that the parameter t can take on all values which produce real numbers for x and y.

EX3) Eliminating the Parameter: Eliminate the parameter and identify the graph of the of the parametric curve

a) $x = 1 - 2t,\ \ y = 2 - t;\ \ -\infty < t < \infty$

b) $x = 3t^2, y = 2t; -2 \le t \le 2$

c) $x = 8\cos t,\ \ y = 6\sin t\ ; 0 \le t \le 2\pi$

d) $x = 2t - 4,\ y = 3t^2$

EX 4) Find the rectangular equation for the following parametric equations.

a) $x = 4\sqrt{t}, \;\; y = t - 3; \;\; -\infty < t < \infty$

b) $x = 1 + 3\cos t, y = 2 + 3\sin t; \;\; 0 \le t \le 2\pi$

c) $x = 4 + 3\sec t, \;\; y = 1 + 2\tan t\,; 0 \le t \le 2\pi$

Ex 5) Finding Parametric Equations for a Line
Find a parameterization of the line through the points $(-2, 3)$ & $(3, 6)$.

EX 6) Simulating Horizontal Motion: Gary walks along a horizontal line (think of it as a number line) with the coordinate of his position (in meters) given by $s = t^3 - 20t^2 + 110t - 85$ where $0 \le t \le 12$. Use parametric equations and a graphing calculator to simulate his motion. Estimate the times when Gary changes direction.

Projectile Motion

Suppose an object is launched vertically from a point s_0 feet above the ground with an initial velocity of v_0 feet per second. The vertical position s (in feet) of the object t seconds after it is launched is

$$s = -16t^2 + v_0 t + s_0.$$

EX 7) Finding Height of a Projectile
A projectile is launched straight up from ground level with an initial velocity of 288 ft /sec.

(a) When is the projectile's height above ground 1152 ft?

(b) When is the projectile's height above ground at least 1152 ft?

EX 8) Simulating Projectile Motion
A distress flare is shot straight up from a ship's bridge 75 ft above the water with an initial velocity of 76 ft/sec. Graph the flare's height against time, give the height of the flare above water at each time, and simulate the flare's motion for each length of time.
(a) 1 sec **(b)** 2 sec **(c)** 4 sec **(d)** 5 sec

Suppose that a baseball is thrown from a point y_0 feet above ground level with an initial speed of v_0 ft/sec at an angle θ with the horizontal. The initial velocity can be represented by the vector $\boldsymbol{v} = \langle \boldsymbol{v_o} \cos \boldsymbol{\theta}, \boldsymbol{v_o} \sin \boldsymbol{\theta} \rangle$
The path of the object is modeled by the parametric equations:

$x =$ ____
[The *x*-component is simply distance = (x-component of initial velocity) × time]

and

y = ____
[The *y*-component is the familiar vertical projectile-motion equation using the *y*-component of initial velocity vector]

EX 9) Hitting a Baseball: Kevin hits a baseball at 3 ft above the ground with an initial speed of 150 ft/sec at an angle of 18° with the horizontal. Will the ball clear a 20-ft wall that is 400 ft away?