

Parametric Equations

Name Key
Date _____

Find each point based on the parametric equations:

1.) $x = 4 - 3t$ and $y = 2t + 5$

a.) $t = 3$

$$\begin{aligned} x &= 4 - 3(3) & y &= 2(3) + 5 \\ x &= -5 & y &= 11 \\ \boxed{(-5, 11)} \end{aligned}$$

b.) $t = -2$

$$\begin{aligned} x &= 4 - 3(-2) = 10 & y &= 2(-2) + 5 = 1 \\ \boxed{(10, 1)} \end{aligned}$$

2.) $x = t^2 + 5t$ and $y = 3 - t^2$

a.) $t = -1$

$$\begin{aligned} x &= (-1)^2 + 5(-1) = -4 & y &= 3 - (-1)^2 = 2 \\ \boxed{(-4, 2)} \end{aligned}$$

b.) $t = 0$

$$\begin{aligned} x &= 0 & y &= 3 \\ \boxed{(0, 3)} \end{aligned}$$

3.) $x = t^3 - 4t$ and $y = \sqrt{t+1}$

a.) $t = 3$

$$\begin{aligned} x &= 3^3 - 4(3) = 15 & y &= \sqrt{3+1} = 2 \\ \boxed{(15, 2)} \end{aligned}$$

b.) $t = 15$

$$\begin{aligned} x &= 15^3 - 4(15) = 3315 & y &= \sqrt{15+1} = 4 \\ \boxed{(3315, 4)} \end{aligned}$$

4.) $x = |t+3|$ and $y = \frac{1}{t}$

a.) $t = -8$

$$\begin{aligned} x &= |-8+3| = 5 & y &= \frac{1}{-8} \\ \boxed{(5, -\frac{1}{8})} \end{aligned}$$

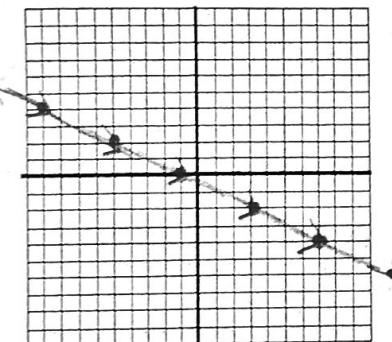
b.) $t = \frac{1}{2}$

$$\begin{aligned} x &= |\frac{1}{2}+3| = 3.5 & y &= \frac{1}{\frac{1}{2}} = 2 \\ \boxed{(3.5, 2)} \end{aligned}$$

For each below, fill in the table, then graph the parametric equations:

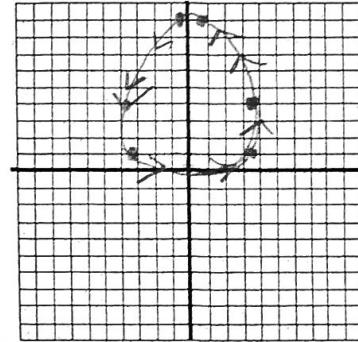
5.) $x(t) = 4t - 1$ $y(t) = -2t$

t	x	y
-3	-13	6
-2	-9	4
-1	-5	2
0	-1	0
1	3	-2
2	7	-4
3	11	-6



7.) $x(t) = 4 \sin(t)$ $y(t) = t^2$

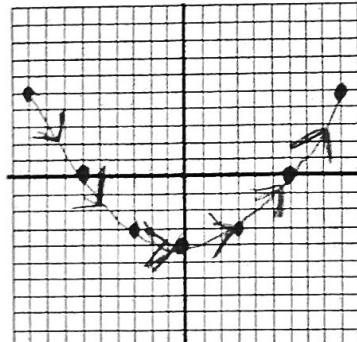
t	x	y
-3	0.56	9
-2	-3.6	4
-1	-3.4	1
0	0	0
1	3.4	1
2	3.6	4
3	0.56	9



6.) $x(t) = 3t$

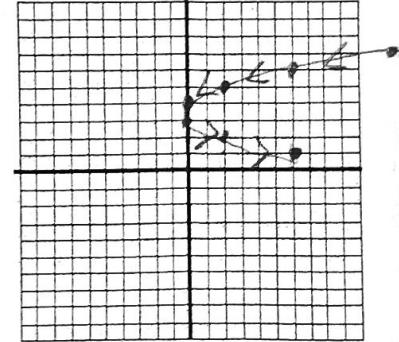
$y(t) = t^2 - 4$

t	x	y
-3	-9	5
-2	-6	0
-1	-3	-3
0	0	-4
1	3	-3
2	6	0
3	9	5



8.) $x(t) = t^2 - t$ $y(t) = 4 - t$

t	x	y
-3	12	7
-2	6	6
-1	2	5
0	0	4
1	0	3
2	2	2
3	6	1



Eliminating the Parameter

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Eliminate the parameter in each set of equations. Your final answer should be fully simplified and in rectangular form (all x's and y's, no t's).

Remember that you will occasionally need to use $\sin^2(x) + \cos^2(x) = 1$

$$1.) \begin{aligned} x(t) &= 5 + t \\ y(t) &= 3t + 1 \end{aligned}$$

$$y = 3(x - 5) + 1$$

$$y = 3x - 15 + 1$$

$$\boxed{y = 3x - 14} \quad \begin{array}{l} \text{Line} \\ \text{slope} = 3 \quad y\text{-int}(0, -14) \end{array}$$

$$5.) \begin{aligned} x(t) &= 3 \cos(t) \\ y(t) &= 4 \sin(t) \end{aligned}$$

$$\frac{x}{3} = \cos t$$

$$\frac{y}{4} = \sin t$$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{16} = 1}$$

Ellipse

$$2.) \begin{aligned} x(t) &= 6t - 1 \\ y(t) &= 2t + 11 \end{aligned}$$

$$\frac{x+1}{6} = t$$

$$y = 2\left(\frac{x+1}{6}\right) + 11$$

$$\text{or } \boxed{y = \frac{1}{3}(x+1) + 11} \quad \text{or } \boxed{x - 3y = 34}$$

$$\boxed{y = \frac{1}{3}x + \frac{1}{3} + 11}$$

$$\boxed{y = \frac{1}{3}x + \frac{34}{3}}$$

line

slope = $\frac{1}{3}$ y-int $(0, \frac{34}{3})$

$$6.) \begin{aligned} x(t) &= 8 - t \\ y(t) &= t^2 + 10t - 100 \end{aligned}$$

$$t = 8 - x$$

$$y = (8-x)^2 + 10(8-x) - 100$$

$$y = x^2 - 16x + 64 + 80 - 10x - 100$$

$$\boxed{y = x^2 - 26x + 44}$$

Parabola

$$3.) \begin{aligned} x(t) &= \frac{3}{t} \\ y(t) &= 6t + 1 \end{aligned}$$

$$t = \frac{3}{x}$$

$$\boxed{y = 6\left(\frac{3}{x}\right) + 1}$$

$$\boxed{y = \frac{18+x}{x}}$$

Rational

$$7.) \begin{aligned} x(t) &= 10 \cos(t) \\ y(t) &= 5 \sin(t) \end{aligned}$$

$$\frac{x}{10} = \cos t$$

$$\frac{y}{5} = \sin t$$

$$\boxed{\frac{x^2}{100} + \frac{y^2}{25} = 1}$$

Ellipse

$$4.) \begin{aligned} x(t) &= \sqrt{t-7} \\ y(t) &= t^2 + 9 \end{aligned}$$

$$\text{or } y = (x^2 + 7)^2 + 9$$

$$\text{or } y = x^4 + 14x^2 + 58$$

Quartic

$$8.) \begin{aligned} x(t) &= \frac{1}{t} \\ y(t) &= \frac{4}{t} \end{aligned}$$

$$y = \frac{4}{x}$$

linear

$$\boxed{y = 4x} \quad x \neq 0$$

$$9.) \begin{aligned} x(t) &= 2 + 5 \sec t \\ y(t) &= 1 + 3 \tan t \end{aligned}$$

$$\frac{x-2}{5} = \sec t$$

$$\frac{y-1}{3} = \tan t$$

$$\boxed{\frac{\sec^2 t - \tan^2 t - 1}{25} = \frac{(y-1)^2}{9} - 1}$$

hyperbola