

## Parametric Equations Applications

1. Nolan Ryan throws a baseball with an initial speed of 145 feet per second at an angle of 20 degrees to the horizontal. The ball leaves Nolan Ryan's hand at a height of 5 feet.

a. Find the parametric equations that describe the position of the ball as a function of time.

$$x = 145 \cos 20^\circ t \quad y = -\frac{1}{2}(32)t^2 + 145 \sin 20^\circ t + 5$$

b. How long is the ball in the air?

$$\approx 3.197 \text{ sec}$$

c. When is the ball at its maximum height?

$$\approx 1.55 \text{ sec}$$

d. What is the maximum height?

$$\approx 43.42 \text{ ft.}$$

e. Determine the distance that the ball traveled.  $x(\tilde{3}.197) = 145 \cos(20^\circ)(\tilde{3}.197)$

$$\approx \boxed{435.65 \text{ ft.}}$$

2. A dart is thrown upward with an initial velocity of 58 feet per second at an angle of elevation  $41^\circ$ . Consider the position of the dart at any time  $t$  ( $t = 0$  when the dart is thrown.) Neglect air resistance.

a. Find parametric equations that model the problem situation.

$$x(t) = 58 \cos(41^\circ)t \quad y(t) = -16t^2 + 58 \sin(41^\circ)t$$

b. When will the dart hit the ground?

$$\approx 2.378 \text{ sec}$$

c. Find the maximum height of the dart. At what time will the dart reach maximum height?

$$\text{max height } 22.62 \text{ ft. @ } 1.189 \text{ sec}$$

d. How far does the dart travel in the horizontal direction? Neglect air resistance.

$$\boxed{104.10 \text{ ft}}$$

3. Suppose that Adam throws a tennis ball off a cliff 300 meters high with an initial speed of 40 meters per second at an angle of 45 degrees to the horizontal.

a. Find the parametric equations that describe the position of the ball as a function of time.

$$x = 40 \cos(45^\circ)t \quad y = -4.9t^2 + 40 \sin(45^\circ)t + 300$$

b. How long is the ball in the air?

$$11.225 \text{ sec}$$

c. How far did the ball travel?

$$317.44 \text{ ft.}$$

4. The center field fence in a ball park is 10 feet high and 400 feet from home plate. A baseball is hit 3 feet above the ground. It leaves the bat at an angle of  $\theta$  degrees with the horizontal at a speed of 100 miles per hour.  $146.6 \text{ ft./sec}$

a. Find the parametric equations that simulate the path of the baseball.

$$x = 146.6 \cos(\theta)t \quad y = -16t^2 + 146.6 \sin(\theta)t + 3$$

b. Determine if a baseball hit at a 15 degree angle is a home run. SHOW ALL WORK!

No, not far enough. The ball drop to a height of 10ft 307.56ft. away from the plate.

c. Determine if a baseball hit at a 23 degree angle is a home run. SHOW ALL WORK!

yes, the ball drops to a height of 10ft, 466.46ft away from the plate

5. Ben can sprint at the rate of 24 feet per second. Jerry sprints at 20 feet per second. Ben gives Jerry a 10 foot head start. They run the 100 yard dash.

$$100 \text{ yds} = 300 \text{ ft.}$$

a. Write two parametric equations that simulate the dash.

$$x_B = 24t \quad x_J = 20t + 10$$

$$y_B = 1 \quad y_J = 2$$

b. Who is ahead in the race after 3 seconds and by how much?

Ben by 2ft.

c. Who wins the race? How many feet has the other runner run when the winner crosses the finish line?

$$B = 12.5 \text{ sec to run } 300 \text{ ft} \quad J = 14.5 \text{ sec to run } 300 \text{ ft.}$$

Ben wins. Jerry ran 260ft at 12.5sec.  $\begin{cases} x = 80 - .7t \\ y = .3t \end{cases}$

6. A hiker in the woods travels along the path described by the parametric equations  $\begin{cases} x = .2t \\ y = 20 + .1t \end{cases}$ . A bear leaves another area of the woods to the west and travels along the path described by the parametric equations  $\begin{cases} x = 80 - .7t \\ y = .3t \end{cases}$ .

Rectangular hiker  $y = -\frac{3}{7}(x - 80)$  bear  $y = 20 + .5x$

a. Do the pathways of the hiker and the bear intersect? Show all work!

yes, the intersect at (15.385, 27.692)

b. Do the hiker and bear collide? Show all work! No, they cross at different times.

hiker  $\approx 92.3 \text{ sec}$

Bear  $\approx 76.92 \text{ sec}$

7. In a pumpkin tossing contest in Morton, Illinois, a contestant won the catapult competition by using two telephone poles, rubber bands, and a power winch. Suppose the pumpkin was launched with an initial speed of 125 feet per second, at an angle of  $40^\circ$ , and from an initial height of 23 feet.

a. Find the parametric equations for the motion of the pumpkin.

$$x = 125 \cos 40^\circ t \quad y = -16t^2 + 125 \sin 40^\circ t + 23$$

b. How far did the pumpkin travel?

$$t = 5.29$$

$$506.87 \text{ ft}$$