Pre-Calc

Notes 9.1 – Introduction to Sequences

Definition: A ________________________ is an ordered progression of numbers. This progression can be

___________________ (meaning it ends), for example{3, 6, 9, 12, …, 21}. Or, it can be _______________, for example {3, 6, 9, 12, ….}.

Notation: a_n is used to denote a term in a sequence. The *a* alone actually has _________________, however the n has a very significant meaning. It indicates the _________ of the term in the sequence being referred to.

There are 2 ways to define these sequences

____________________ & ____________________

The **explicit definition** is like a formula.

Ex1) Find the first four terms of the given sequence.

a) $a_n = 2n + 3$ **b)** $a_n = 3 \cdot 2^n$ **c)** $a_n = n + \frac{1}{n}$

____, ____, ____, ____ (a_1, a_2, a_3, a_4) ____, ____, ____, ____ (a_1, a_2, a_3, a_4) ____, ____, ____, ____ (a_1, a_2, a_3, a_4)

NOW YOU TRY ☺ Find the first four terms of the given sequence.

a) $a_n = n^3 + 1$ **b)** $a_n = 3 - 7n$ **c)** $a_n = (-2)^n$

____, ____, ____, ____ (a_1, a_2, a_3, a_4) ____, ____, ____, ____ (a_1, a_2, a_3, a_4) ____, ____, ____, ____ (a_1, a_2, a_3, a_4)

The **recursive definition** has 2 parts:

(1) a term to begin with
(2) a symbolic description of how the successive terms are related.

Ex2) Find the indicated terms of the given sequence.

a) $a_1 = 6$, $a_n = 4 + a_{n-1}$ **b)** $a_1 = 9$, $a_n = \frac{1}{3} \cdot a_{n-1}$ **c)** $a_1 = 1, a_2 = 2$, $a_n = a_{n-1} + a_{n-2}$

____, ____, ____, ____ (a_2, a_3, a_4, a_5) ____, ____, ____, ____ (a_2, a_3, a_4, a_5) ____, ____, ____, ____ (a_2, a_3, a_4, a_5)

NOW YOU TRY ☺ Find the indicated terms of the given sequence.

a) $a_1 = 4$, $a_n = 5 \cdot a_{n-1} + 2$ **b)** $a_1 = 1$, $a_n = \left(-\frac{1}{3}\right)^n \cdot a_{n-1}$ **c)** $a_1 = 1, a_2 = 2$, $a_n = a_{n-1} \cdot a_{n-2}$

____, ____, ____, ____ (a_2, a_3, a_4, a_5) ____, ____, ____, ____ (a_2, a_3, a_4, a_5) ____, ____, ____, ____ (a_2, a_3, a_4, a_5)

Although it is possible to work with many different types of sequences, there are 2 that are most common.

___________________ (where there is a common difference between each term) and ___________________ (where there is a common ration between each pair of terms).

ARITHMETIC:

$a_n = a_1 + d(n-1)$, were d is the difference between each term (called the common difference)

Ex3) State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the *n*th term of the sequence in terms of *n*.

a) 17, 21, 25, 29, … **b)** 8, 12, 18, 27, … **c)** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$ **d)** 11, 101, 1001, 10001, …

type: ____________ type: ____________ type: ____________ type: ____________

a_n = _____________ a_n = _____________ a_n = _____________ a_n = _____________

NOW YOU TRY ☺ State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the *n*th term of the sequence in terms of *n*.

a) 100, –50, 25, –12.5,… **b)** 1, 4, 9, 16, … **c)** $\frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots$ **d)** 2a – 2b, 3a – b, 4a, 5a + b, …

type: ____________ type: ____________ type: ____________ type: ____________

a_n = _____________ a_n = _____________ a_n = _____________ a_n = _____________

Ex4) State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the *n*th term of the sequence in terms of *n*.

a) $a_1 = 8,\ a_n = \frac{1}{2} \cdot a_{n-1}$ **b)** $a_1 = 6,\ a_n = a_{n-1} + 10$ **c)** $a_1 = \frac{1}{2},\ a_n = \frac{n}{n+1}(a_{n-1} + 1)$

type: ____________ type: ____________ type: ____________

a_n = _____________ a_n = _____________ a_n = _____________

NOW YOU TRY ☺ State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the *n*th term of the sequence in terms of *n*.

a) $a_1 = 1,\ a_n = a_{n-1} + 2n - 1$ **b)** $a_1 = 3,\ a_n = -2 \cdot a_{n-1}$ **c)** $2^{\frac{2}{3}}, 2^{\frac{5}{3}}, 2^{\frac{8}{3}}, \ldots$

type: ____________ type: ____________ type: ____________

a_n = _____________ a_n = _____________ a_n = _____________

Ex5) Find the indicated term of each arithmetic sequence:

a) $a_1 = 15,\ a_2 = 21,\ a_{20} = ?$

b) $a_1 = 15,\ a_2 = 21,\ a_{20} = ?$

Ex6) How many terms are in the finite arithmetic sequence

a) 18, 24, …, 336

b) 178, 170, …, 2

Ex7) Find the number of multiples of …

a) 7 between 30, and 300.

b) 6 between 28, and 280.

Ex 8) Find the

a) 100th term of the sequence
15, 12.3, 9.6, 6.9….

b) 120th term of the sequence
-4, 2, 8, 14 ….

Ex9) Find the explicit definition for the sequences below:

a) $\frac{2}{5}, \frac{11}{15}, \frac{16}{15}, \frac{7}{5}, \cdots$

b) $\frac{7}{6}, \frac{5}{3}, \frac{13}{6}, \frac{8}{3}, \cdots$

c) -10, -6,-2, 2, . . .

d) -10.3, -6.5, -2.7, 1.1, . . .

Ex10)

a) Which term in the sequence 1, 4, 7,….is 88?

b) Which term in the sequence 1, 5, 9,….is 181?