9.3 Arithmetic Series

Sequence $1, 2, 3, 4, \ldots$

Series $1 + 2 + 3 + 4 + \cdots + 100$

$$\begin{array}{r} 1 + 2 + 3 + \cdots + 98 + 99 + 100 \\ + \; 100 + 99 + 98 + \cdots + 3 + 2 + 1 \\ \hline 101 + 101 + 101 \end{array}$$

Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$\frac{100}{2}(101)$$

$$50(101) = 5050$$

Ex) $\underset{a_1}{1} + 3 + 5 + 7 + \cdots + \underset{a_{25}}{49}$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{25} = \frac{25}{2}(1 + 49)$$

$$S_{25} = \frac{25}{2}(50)^{25}$$

$$S_{25} = 625$$

$49 = 1 + 2(n-1)$

$48 = 2(n-1)$

$24 = n - 1$

$25 = n$

Ex) $\sum_{n=1}^{6} (2n+1)$ ← Sigma Notation

Explicit Formula

Sum

$a_1 = 2(1) + 1 = 3$

$a_6 = 2(6) + 1 = 13$

$$S_6 = \frac{n}{2}(a_1 + a_6)$$

$$S_6 = \frac{6}{2}(3 + 13)$$

$$= 3(16)$$

$$= 48$$

Ex) $\sum_{n=3}^{7} (4n+5)$

$a_3 = 4(3) + 5 = 17$

$a_7 = 4(7) + 5 = 33$

$$S_n = \frac{n}{2}(a_3 + a_7)$$

$$S_5 = \frac{5}{2}(17 + 33)$$

$$= \frac{5}{2}(50)^{25} = 125$$