Pre-Calculus

Name Key ID: 1

Geometric Series

Date _____ Period ____

Evaluate each geometric series described.

1) $2 - 12 + 72 - 432..., \ n = 7$

$S_7 = \frac{2(1-(-6)^7)}{1-(-6)} = 119973$

2) $-1 - 5 - 25 - 125..., \ n = 9$

$S_9 = \frac{-1(1-5^9)}{(1-5)} = -488281$

3) $-1 + 4 - 16 + 64..., \ n = 6$

$S_6 = \frac{-1(1-(-4)^6)}{1-(-4)} = 819$

4) $-4 - 20 - 100 - 500..., \ n = 6$

$S_6 = \frac{-4(1-5^6)}{1-5} = -15624$

5) $\sum_{i=1}^{7} -2 \cdot (-5)^{i-1} = \frac{-2(1-(-5)^7)}{1-(-5)} = -26042$

6) $\sum_{i=1}^{8} 2^{i-1} = \frac{1(1-2^8)}{1-2} = 255$

7) $\sum_{k=1}^{9} 3^{k-1} = \frac{1(1-3^9)}{1-3} = 9841$

8) $\sum_{m=1}^{8} 3\,2^{m-1} = \frac{3(1-2^8)}{1-2} = 765$

9) $\sum_{k=1}^{7} 3^{k-1} = \frac{1(1-3^7)}{1-3} = 1093$

10) $\sum_{m=1}^{10} 4^{m-1} = \frac{1(1-4^{10})}{1-4} = 349525$

Determine the number of terms *n* in each geometric series.

11) $a_1 = 4, \ r = -5, \ S_n = -10416$

$-10416 = \frac{4(1-(-5)^n)}{1-(-5)}$

$n = 6$ 6 terms

12) $a_1 = -1, \ r = 2, \ S_n = -7$

$-7 = \frac{-1(1-2^n)}{1-2}$

$7 = -1(1-2^n)$

$-7 = (1-2^n)$

$-8 = -2^n$

$8 = 2^n$

$2^3 = 2^n$

$n = 3$

3 terms

13) $a_1 = -4, \ r = -6, \ S_n = -159964$

$-159964 = \frac{-4(1-(-6)^n)}{1-(-6)}$

$\frac{-7}{4} \cdot -159964 = \frac{-4(1-(-6)^n)}{7} \cdot \frac{-7}{4}$

$279937 = (1-(-6)^n)$

$-279936 = (-6)^n$

$\ln 279936 = n \ln 6$

$n = 7$

7 terms

14) $a_1 = 1, \ r = 3, \ S_n = 121$

$121 = \frac{1(1-3^n)}{1-3}$

$121 = \frac{1(1-3^n)}{-2}$

$243 = 3^n$

$3^5 = 3^n \quad n = 5$

5 terms

Determine if each geometric series converges or diverges.

15) $\frac{32}{81} - \frac{16}{27} + \frac{8}{9} - \frac{4}{3} \ldots$ diverge $r = 3/2$

16) $\frac{7}{6} + \frac{7}{18} + \frac{7}{54} + \frac{7}{162} \ldots$ $r = \frac{1}{3}$ converge

17) $1 + 4 + 16 + 64 \ldots$ diverge

18) $4 - 8 + 16 - 32 \ldots$ diverge

19) $-6 - 3 - \frac{3}{2} - \frac{3}{4} \ldots$ converge

20) $\frac{135}{8} + \frac{45}{4} + \frac{15}{2} + 5 \ldots$ diverge

21) $\sum_{n=1}^{\infty} -64 \cdot \left(\frac{1}{2}\right)^{n-1}$ converge

22) $\sum_{k=1}^{\infty} -4 \cdot \left(\frac{1}{5}\right)^{k-1}$ converge

Evaluate each infinite geometric series described.

23) $1 + 0.6 + 0.36 + 0.216 \ldots$

$\sum_{n=1}^{\infty} 1(.6)^{n-1} = \frac{1}{1-.6} = 2.5$

24) $-250 + 50 - 10 + 2 \ldots$ $S = \frac{-250}{\left(1 - -\frac{1}{2}\right)} = -166\frac{2}{3}$

25) $1 - 2 + 4 - 8 \ldots$

$S = \frac{1}{1 - -2} = \frac{1}{3}$

26) $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} \ldots$ $S = \frac{1}{1 - 3/4} = 4$

27) $\sum_{i=1}^{\infty} 0.8^{i-1}$ $S = \frac{1}{1-.8} = 5$

28) $\sum_{n=1}^{\infty} -\frac{81}{16} \cdot \left(\frac{2}{3}\right)^{n-1}$ $S = \frac{-\frac{81}{16}}{1 - 2/3}$

$= -\frac{81}{16} \cdot 3 = \frac{-243}{16}$

29) $\sum_{k=1}^{\infty} \frac{3}{2} \cdot \left(\frac{1}{4}\right)^{k-1}$ $S = \frac{\frac{3}{2}}{1 - \frac{1}{4}}$

$\frac{3}{2} \div \frac{3}{4} = \frac{3}{2} \cdot \frac{4}{3} = 2$

30) $\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{k-1}$

$S = \frac{1}{1 - 1/3} = \frac{3}{2}$

Determine the common ratio of the infinite geometric series.

31) $a_1 = -4.2,\ S = -21$

$-21 = \frac{-4.2}{1-r}$

$1 - r = \frac{-4.2}{-21}$

$1 - r = .2$

$-r = -.8$

$r = .8$

32) $a_1 = 4,\ S = 8$

$8 = \frac{4}{1-r}$

$1 - r = \frac{4}{8}$

$-r = -\frac{1}{2}$

$r = \frac{1}{2}$

33) $a_1 = 1,\ S = \frac{2}{3}$

$\frac{2}{3} = \frac{1}{1-r}$

$3 = 2(1-r)$

$\frac{3}{2}$

$-\frac{1}{2} = r$

34) $a_1 = 1,\ S = \frac{4}{3}$

$S = \frac{a_1}{1-r}$

$\frac{4}{3} = \frac{1}{1-r}$

$3 = 4 - 4r$

$-1 = -4r$

$r = \frac{1}{4}$