Sequences & Series Review

① $a_n = \frac{n}{n+1}$

$a_1 = \frac{1}{1+1} = \frac{1}{2}$

$a_2 = \frac{2}{2+1} = \frac{2}{3}$

$a_3 = \frac{3}{3+1} = \frac{3}{4}$

$a_4 = \quad = \frac{4}{5}$

$a_5 = \quad = \frac{5}{6}$

$\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\}$

② $a_n = ne + n$

$a_1 = e + 1$

$a_2 = 2e + 2$

$a_3 = 3e + 3$

$a_4 = 4e + 4$

$a_5 = 5e + 5$

$\{e+1, 2e+2, 3e+3, 4e+4, 5e+5\}$

③ $a_{n+1} = 4a_n$

$a_1 = 2$

$a_2 = 4(2) = 8$

$a_3 = 4(8) = 32$

$a_4 = 4(32) = 128$

$a_5 = 5(32) = 160$

$\{2, 8, 32, 128, 160\}$

④ $a_n = (-1)^{n-1}(2n)$

$a_1 = (-1)^{1-1}(2(1)) = 2$

$a_2 = (-1)^{2-1}(2(2)) = -4$

$a_3 = (-1)^{3-1}(2(3)) = 6$

$a_4 = (-1)^{4-1}(2(4)) = -8$

$a_5 = (-1)^{5-1}(2(5)) = 10$

$\{2, -4, 6, -8, 10\}$

⑤ $a_n = -2a_{n+1} - 7 \quad a_1 = -2$

$a_n + 7 = -2a_{n+1}$

$-\frac{1}{2}a_n - \frac{7}{2} = a_{n+1}$

$a_{n+1} = -.5a_n - 3.5$

Since a_n comes before a_{n+1} I solved for a_{n+1}.

$a_{n+1} = -.5a_n - 3.5$

$a_1 = -2$

$a_2 = -.5(-2) - 3.5 = -2.5$

$a_3 = -.5(-2.5) - 3.5$

$a_4 = -.5(\quad) - 3.5$

$a_5 = -.5(\quad) - 3.5$

⑥ $a_n = \frac{n^2}{1+n}$

$a_1 = \frac{1^2}{1+1} = \boxed{\frac{1}{2}}$

$a_2 = \frac{2^2}{1+2} = \boxed{\frac{4}{3}}$

$a_3 = \frac{3^2}{1+3} = \boxed{\frac{9}{4}}$

$a_4 = \frac{4^2}{1+4} = \boxed{\frac{16}{5}}$

$a_5 = \frac{5^2}{1+5} = \boxed{\frac{25}{6}}$

$\{\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}\}$

⑦ $1, -4, 9, -16, \ldots$

$\boxed{\{(-1)^{n-1} n^2\}}$

⑧ $-8, -5, -2, 1, 4, \ldots$

$+3 \quad +3$

arithmetic

$\boxed{a_n = -8 + 3(n-1) \text{ or } a_n = -11 + 3n}$

⑨ $3, -6, 12, -24$

$r = -2 \quad -2$

geometric

$\boxed{a_n = 3(-2)^{n-1} \text{ or } a_n = -1.5(-2)^n}$

(10) $a_9 = 18$ $a_{53} = 7$

$a_n = a_1 + d(n-1)$

$a_{53} = a_9 + d(53-9)$

$7 = 18 + d(44)$

$\frac{-11}{44} = \frac{\cancel{46}d}{\cancel{46}}$

$d = -\frac{1}{4}$

$\boxed{a_n = 18 - \frac{1}{4}(n-9)}$

or

$\boxed{a_n = 7 - \frac{1}{4}(n-53)}$

or

$\boxed{a_n = 20 - \frac{1}{4}(n-1)}$

(11) $a_{10} = -40$ $a_{82} = 176$

$a_{82} = a_{10} + d(82-10)$

$176 = -40 + d(72)$

$\frac{216}{72} = \frac{d(72)}{72}$

$\boxed{3 = d}$

$\boxed{a_n = -40 + 3(n-10)}$

or

$\boxed{a_n = 176 + 3(n-82)}$

or

$\boxed{a_n = -67 + 3(10-1)}$

(12) $a_4 = -20$ $a_{10} = \frac{-4}{3125}$

$a_{10} = a_4 (r)^{(10-4)}$

$\frac{1}{\cancel{20}} \cdot \frac{-\cancel{4}}{3125} = \frac{-20(r)^6}{-20}$

$\left(\frac{1}{15625}\right)^{1/6} = \left(r^6\right)^{1/6}$

$.2 = r$

$a_n = a_1 (.2)^{n-1}$

$-20 = a_1 (.2)^{4-1}$

$\frac{-20}{.2^3} = \frac{a_1 (.2)^3}{.2^3}$

$-2500 = a_1$

$\boxed{a_n = -20(.2)^{n-4}}$

or

$\boxed{a_n = \frac{-4}{3125}(.2)^{n-10}}$

or

$\boxed{a_n = -2500(.2)^{n-1}}$

(13) $a_3 = 81$ $a_9 = \frac{1}{9}$

$a_9 = a_3 (r)^{9-3}$

$81 = \frac{1}{9}(r)^6$

$(729)^{1/6} = (r^6)^{1/6}$

$r = 3$

$\boxed{a_n = 81(3)^{n-3}}$

or

$\boxed{a_n = \frac{1}{9}(3)^{n-9}}$

or

$\boxed{a_n = 729(3)^{n-1}}$

(14) $\sum_{k=1}^{5} 10 = 10+10+10+10+10 = 5(10) = \boxed{50}$

(15) $\sum_{m=2}^{7} 2m+4$ ← Linear... so arithmetic series

use $S_n = \frac{n}{2}(a_1 + a_n)$

$a_2 = 2(2)+4 = 8$

$a_7 = 2(7)+4 = 18$

$S_n = \frac{6}{2}(a_2 + a_7)$

$S_n = 3(8+18)$

$S_n = \boxed{78}$

(16) $\sum_{i=-2}^{3} (i^2+i) = (-2)^2+(-2) = 2$

$= (-1)^2+(-1) = 0$

$= (0)^2+(0) = 0$

$= (1)^2+(1) = 2$

$= (2)^2+(2) = 6$

$+ \boxed{10}$

(17) $\sum_{n=0}^{\infty} \left[5\left(\frac{2}{3}\right)^n\right]$

Exponential

Infinite So, geometric

start

$S = \frac{a_0}{1-r}$

$S = \frac{5}{1-2/3}$

$= 5 \div 1/3$

$= \boxed{15}$

Since Infinite

(18) $S = \frac{a_3}{1-r}$

$a_3 = 4(\frac{1}{3})^3$

$= 4\frac{1}{27}$

$= \frac{4}{27}$

$S = \frac{\frac{4}{27}}{1-\frac{1}{3}}$

$= \frac{4}{27} \div \frac{2}{3}$

$= \frac{\cancel{4}\,2}{\cancel{27}\,9} \cdot \frac{\cancel{3}}{\cancel{2}}$

$= \boxed{\frac{2}{9}}$

(19) Linear so arithmetic

So the sum just gets larger. $\boxed{\infty}$

20

(20) $1+4+7+\cdots+82$

$\underset{+3}{\vee}\quad\underset{+3}{\vee}$ arithmetic $d=3$

$a_n = a_1 + d(n-1)$

$82 = 1 + 3(n-1)$

$81 = 3(n-1)$

$27 = n-1$

$28 = n$

$$\sum_{n=1}^{28} 1+3(n-1)$$

or simplify

or

$$\boxed{\sum_{n=1}^{28} 3n-2}$$

(21) $23+17+11+\cdots-229$

$\underset{-6}{\vee}\quad\underset{-6}{\vee}$ arithmetic $d=-6$

$a_n = 29-6n$

$-229 = 29-6n$

$-258 = -6n$

$43 = n$

$$\boxed{\sum_{n=1}^{43} (29-6n)}$$

(22) $5+15+45+\cdots+3645$

$\underset{\times 3}{\vee}\quad\underset{\times 3}{\vee}$ geometric $r=3$

$a_n = 5(3)^{n-1}$

$3645 = 5(3)^{n-1}$

$729 = 3^{n-1}$

$3^6 = 3^{n-1}$

$6 = n-1$

$7 = n$

$$\boxed{\sum_{n=1}^{7} 5(3)^{n-1}}$$

(23) $2+\frac{1}{2}+\frac{1}{8}+\cdots+\frac{1}{2048}$

$\underset{\cdot\frac{1}{4}}{\vee}\quad\underset{\cdot\frac{1}{4}}{\vee}$ geometric $r=\frac{1}{4}$

$$\boxed{\sum_{n=1}^{7} 2\left(\tfrac{1}{4}\right)^{n-1}}$$

$\frac{1}{2048} = 2\left(\frac{1}{4}\right)^{n-1}$

$\frac{1}{4096} = \left(\frac{1}{4}\right)^{n-1}$

$\frac{\ln\left(\frac{1}{4096}\right)}{\ln(1/4)} = \frac{(n-1)\ln\left(\frac{1}{4}\right)}{\ln(1/4)}$

$6 = n-1$

$n = 7$

(24) $1-4+9-16+25+\cdots+121$

$$\boxed{\sum_{n=1}^{11} (-1)^{n-1} n^2}$$

(25) Since arithmetic

$$S_n=\frac{n}{2}(a_1+a_n)$$

$$4047=\frac{n}{2}(15+a_n)$$

$$a_n=a_1+d(n-1)$$

$$a_n=15+2(n-1)$$

$$a_n=13+2n$$

$$4047=\frac{n}{2}(15+13+2n)$$

$$4047=\frac{n}{2}(28+2n)$$

$$0=n^2+14n-4047$$

pos. $n=57$

(26) $S_n=315 \quad a_1=315 \quad d=-9$

$$S_n=\frac{n}{2}(a_1+a_n)$$

$$315=\frac{n}{2}(315+a_n)$$

$$a_n=a_1+d(n-1)$$

$$a_n=315-9(n-1)$$

$$a_n=324-9n$$

$$315=\frac{n}{2}(315+324-9n)$$

$$315=\frac{n}{2}(639-9n)$$

$$0=-4.5n^2+319.5n-315$$

positive $\boxed{n=70}$

(27) $S_n=-32767 \quad a_1=-1 \quad r=2$ ← geometric

$$S_n=\frac{a_1(1-r^n)}{1-r}$$

$$-32767=\frac{\cancel{-1}(1-2^n)}{\cancel{1-2}}$$

$$-32767=-2^n$$

$$32767=2^n$$

$$\frac{\ln(32767)}{\ln 2}=\frac{n\ln 2}{\ln 2}$$

$$\boxed{n=15}$$

(28)

$$S_n=\frac{a_1(1-r^n)}{1-r}$$

$$531440=\frac{2(1-3^n)}{1-3}$$

$$531440=\frac{\cancel{2}(1-3^n)}{-\cancel{2}}$$

$$-531440=1-3^n$$

$$-531441=-3^n$$

$$531441=3^n$$

$$\frac{\ln(531441)}{\ln 3}=\frac{n\ln 3}{\ln 3} \quad \boxed{n=12}$$

(29) $a_1=16 \quad a_2=48 \quad a_3=80$

$+32 \quad +32 \quad d=32$

$$a_6=16+32(6-1)=176$$

$$S_6=\frac{6}{2}(16+176)=\boxed{576\text{ ft.}}$$

(30) $a_3 = 180°$ $a_4 = 360°$ $a_5 = 540°$

$+180$ $+180$ $d = 180$

$$a_n = a_3 + d(n-3)$$

$$a_{50} = 180 + 180(50-3)$$

$$\boxed{a_{50} = 8640°}$$

(31) $\uparrow 6\%$

$r = 1.06$ $a_0 = 500$

$$a_n = 500(1.06)^{\frac{n}{2}} \leftarrow \text{every 2 hours}$$

$$a_{24} = 500(1.06)^{\frac{24}{2}} \approx \boxed{1006 \text{ bacteria}}$$

(32) $a_1 = 2500$

$r = 1 - .45$

$r = .55$

$$a_n = a_0(r)^n$$

$$\boxed{a_n = 2500(.55)^n}$$

(33) 1 ton = 2000 pounds $r = 1 - \frac{1}{5}$

$r = \frac{4}{5}$

$$a_n = a_0(r)^n$$

$$a_n = 2000\left(\frac{4}{5}\right)^n$$

$$a_5 = 2000\left(\frac{4}{5}\right)^5$$

$$a_5 = \boxed{655.36 \text{ lbs}}$$

(34) $(2x^3 - 3)^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$1(2x^3)^4(-3)^0 + 4(2x^3)^3(-3)^1 + 6(2x^3)^2(-3)^2 + 4(2x^3)^1(-3)^3 + 1(2x^3)^0(-3)^4$$

$$\boxed{16x^{12} - 96x^9 + 216x^6 - 216x^3 + 81}$$

(35) $(3x^3 + 4)^6$

$$1(3x^3)^6(4)^0 + 6(3x^3)^5(4)^1 + 15(3x^3)^4(4)^2 + 20(3x^3)^3(4)^3 + 15(3x^3)^2(4)^4 + 6(3x^3)^1(4)^5 + 1(3x^3)^0(4)^6$$

$$\boxed{712x^{18} + 5832x^{15} + 19440x^{12} + 34560x^9 + 34560x^6 + 18432x^3 + 4096}$$

7th term

36 $_9C_4$

$_9C_6(y)^3(2)^6$

$84y^3 \cdot 64$

$\boxed{5376y^3}$

5376

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\
1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \\
1y^9 \quad 9y^8 \quad 36y^7 \quad 84y^6 \quad 126y^5 \quad 126y^4 \quad \boxed{84}y^3 \quad 36y^2 \quad 9y^1 \quad 1y^0 \\
1 \quad 10 \quad 44 \quad 120 \quad 210 \quad 252 \quad 210 \quad 4 \quad 44 \quad 10 \quad 1
\end{array}
$$

37 same as 36

38

$_{10}C_4(2a)^6(-3b)^4$

$210 \cdot 64a^6 \cdot 81b^4$

$\boxed{1128960}a^6b^4$

39 $(x^2+4y)^9$ 5th term

$_9C_4(x^2)^5(4y)^4$

$126\,x^{10} \cdot 256y^4$

$\boxed{32256x^{10}y^4}$