Final Exam Review

Point on unit circle

① $(3, 315^\circ)$

$(r\cos\theta, r\sin\theta)$

$(3\cos 315^\circ, 3\sin 315^\circ)$

$\left(3 \cdot \frac{\sqrt{2}}{2}, 3 \cdot -\frac{\sqrt{2}}{2}\right)$

$\boxed{\left(\frac{3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)}$

② $(2, \frac{\pi}{3})$

$(2\cos\frac{\pi}{3}, 2\sin\frac{\pi}{3})$

$(2 \cdot \frac{1}{2}, 2 \cdot \frac{\sqrt{3}}{2})$

$\boxed{(1, \sqrt{3})}$

③ $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

(r, θ)

$\boxed{(1, \frac{4\pi}{3})}$

④ $(\sqrt{2}, -\sqrt{2})$

$\boxed{(2, \frac{7\pi}{4})}$

$(\sqrt{2})^2 + (-\sqrt{2})^2 = r^2$

$2 + 2 = r^2$

$4 = r^2$

$2 = r$

⑤ $(x+3)^2 + (y-1)^2 = 10$

$x^2 + 6x + \cancel{9} + y^2 - 2y + \cancel{1} = \cancel{10}$

$x^2 + y^2 + 6x - 2y = 0$

$r^2 + 6r\cos\theta - 2r\sin\theta = 0$

$r(r + 6\cos\theta - 2\sin\theta = 0$

$\cancel{r=0}$ $\quad r + 6\cos\theta - 2\sin\theta = 0$

$\boxed{r = -6\cos\theta + 2\sin\theta}$

Polar

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

⑥ $x^2 + (y-2)^2 = 4$

$x^2 + y^2 - 4y + \cancel{4} = \cancel{4}$

$r^2 - 4r\sin\theta = 0$

$r(r - 4\sin\theta) = 0$

$\cancel{r=0}$ $\quad r - 4\sin\theta = 0$

$\boxed{r = 4\sin\theta}$

⑦ $(x-2)^2 + (y+1)^2 = 5$

$x^2 + 4x + \cancel{4} + y^2 + 2y + \cancel{1} = \cancel{5}$

$x^2 + y^2 + 4x + 2y = 0$

$r^2 + 4r\cos\theta + 2r\sin\theta = 0$

$r(r + 4\cos\theta + 2\sin\theta) = 0$

$\cancel{r=0}$ $\quad r + 4\cos\theta + 2\sin\theta = 0$

$\boxed{r = -4\cos\theta - 2\sin\theta}$

⑧ $(x+2)^2 + (y-1)^2 = 5$

$x^2 + 4x + \cancel{4} + y^2 - 2y + \cancel{1} = \cancel{5}$

$x^2 + y^2 + 4x - 2y = 0$

$r^2 + 4r\cos\theta - 2r\sin\theta = 0$

$r(r + 4\cos\theta - 2\sin\theta) = 0$

$\cancel{r=0}$ $\quad r + 4\cos\theta - 2\sin\theta = 0$

$\boxed{r = -4\cos\theta + 2\sin\theta}$

⑨ $r = \frac{2}{3 + 2\cos\theta}$

$r(3 + 2\cos\theta) = 2$

$3r + 2r\cos\theta = 4$

$3r + 2x = 4$

$(3r)^2 = (4 - 2x)^2$

$9r^2 = 16 - 16x + 4x^2$

$9x^2 + 9y^2 = 16 - 16x + 4x^2$

$\boxed{5x^2 + 9y^2 + 16x - 16 = 0}$

Ellipse

(10) $r = -2\sin\theta$

$r^2 = -2r\sin\theta$

$x^2 + y^2 = -2y$

$\boxed{x^2 + y^2 + 2y = 0}$

Circle

(11) $r = \frac{6}{2-\sin\theta}$

$r(2-\sin\theta) = 6$

$2r - r\sin\theta = 6$

$2r - y = 6$

$(2r)^2 = (y+6)^2$

$4r^2 = y^2 + 12y + 36$

$4x^2 + 4y^2 = y^2 + 12y + 36$

$\boxed{4x^2 + 3y^2 - 12y - 36 = 0}$

Ellipse

(12) $r = 2\cos\theta + 6\sin\theta$

$r^2 = 2r\cos\theta + 6r\sin\theta$

$x^2 + y^2 = 2x + 6y$

$\boxed{x^2 + y^2 - 2x - 6y = 0}$

Circle

(13) $x = 5\cos t \quad y = 2\sin t$

$\frac{x}{5} = \cos t \quad \frac{y}{2} = \sin t$

use pyth. ident.

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\boxed{\frac{x^2}{25} + \frac{y^2}{4} = 1}$

Ellipse

(14) $x = \cos t + 1 \quad y = 4\sin t - 1$

$x - 1 = \cos t \quad \frac{y+1}{4} = \sin t$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x-1}{1}\right)^2 + \left(\frac{y+1}{4}\right)^2 = 1$

$\boxed{\frac{(x-1)^2}{1} + \frac{(y+1)^2}{16} = 1}$

Ellipse

(15) $x = -2t - 2 \quad y = t^2 - 4$

$\boxed{\frac{x+2}{-2}} = t$

Substitute in for t

$y = \left(\frac{x+2}{-2}\right)^2 - 4$

$\boxed{y = \frac{1}{4}(x+2)^2 - 4}$

parabola

(16) $x = 2t - 3 \quad y = \frac{2t^2}{3} - 2t + \frac{3}{2}$

$\frac{x+3}{2} = t$

$y = \frac{2\left(\frac{x+3}{3}\right)^2}{3} - 2\left(\frac{x+3}{3}\right) + \frac{3}{2}$

$y = \frac{2}{3} \cdot \frac{(x+3)^2}{9} - \frac{2(x+3)}{3} + \frac{3}{2}$

(17) $x^2 + y^2 - 4y - 60 = 0$

$x^2 + y^2 - 4y + 4 = 60 + 4$

$\boxed{x^2 + (y-2)^2 = 64}$

Circle

center $(0,2)$

radius $= 8$

(18) $9x^2 + 16y^2 + 72x - 320y + 448 = 0$

$9x^2 + 72x + 16y^2 - 320y = -448$

$9(x^2 + 8x + 16) + 16(y^2 - 20y + 100) = -448 + 144 + 1600$

$\frac{9(x+4)^2}{1296} + \frac{16(y-10)^2}{1296} = \frac{1296}{1296}$

$9^2 + c^2 = 12^2$

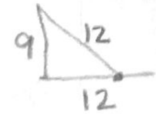
$c^2 = 144 - 81$

$c^2 = 63$

$c = \sqrt{63}$

$c = 3\sqrt{7}$

$\boxed{\frac{(x+4)^2}{144} + \frac{(y-10)^2}{81} = 1}$



Ellipse: center $(-4,10)$ foci $(-4 \pm 3\sqrt{7}, 10)$

Verticies $(8,10)$ & $(-16,10)$

co-verticies $(-4,19)$ $(-4,1)$

major axis $= 24$

minor axis $= 18$

(19) $x^2 - y^2 + 8x - 18y - 114 = 0$

$x^2 + 8x \qquad - y^2 - 18y \qquad = 114$

$x^2 + 8x + 16 - (y^2 + 18y + 81) = 114 + 16 - 81$

$\frac{(x+4)^2}{49} - \frac{(y+9)^2}{49} = \frac{49}{49}$

$$\boxed{\frac{(x+4)^2}{49} - \frac{(y+9)^2}{49} = 1}$$

Hyperbola

center $(-4, -9)$

verticies $(3, -9)$ & $(-11, -9)$

slope of asymptotes $= \pm\frac{7}{7} = \pm 1$

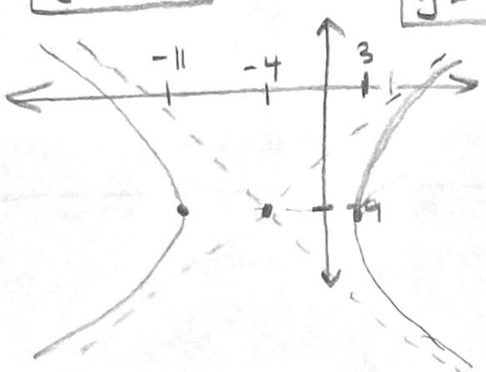
eqns. of asymp

$y + 9 = \pm 1(x+4)$

or

$y + 9 = x + 4$ & $y + 9 = -x - 4$

$\boxed{y = x - 5}$ $\boxed{y = -x - 13}$



(20) $-3x^2 - 60x + 7y - 307 = 0$

$7y = 3x^2 + 60x + 307$

$7y = (3x^2 + 60x \quad) + 307$

$7y = 3(x^2 + 20x + 100) + 307 - 300$

$\frac{7y}{7} = \frac{3(x+10)^2}{7} + \frac{7}{7}$

$$\boxed{y = \frac{3}{7}(x+10)^2 + 1}$$

$\frac{1}{4p} = \frac{3}{7}$

$12p = 7$

$p = \frac{7}{12}$ ← focal length

Vertex $(-10, 1)$

focus $(-10, 1 + \frac{3}{7})$

$(-10, \frac{10}{7})$

directrix

$y = \frac{4}{7}$

(21) $\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{5}{16}$

$\theta = \cos^{-1}\left(\frac{5}{16}\right)$

$$\boxed{\theta = 71.8^\circ}$$

(22) $\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{7}{6}$

$\theta = \tan^{-1}\left(\frac{7}{6}\right)$

$$\boxed{\theta = 49.4^\circ}$$

(23) $\sin 33^\circ = \frac{3}{x}$

$x = \frac{3}{\sin 33^\circ}$

$$\boxed{x \approx 5.5}$$

(25) SAS Law of Cosines

$a^2 = 21^2 + 30^2 - 2(21)(30)\cos 123^\circ$

$\boxed{a \approx 45.0}$

$\frac{\sin 123^\circ}{a} = \frac{\sin B}{21}$

$\boxed{B \approx 23.0^\circ}$

$C = 180^\circ - 123^\circ - 23.0^\circ$

$\boxed{C \approx 34.0^\circ}$

(24) $\tan 71^\circ = \frac{7}{x}$

$x = \frac{7}{\tan 71^\circ}$

$$\boxed{x \approx 2.4}$$

(26) $b^2 = 28^2 + 29^2 - 2(28)(29)\cos 92^\circ$

$\boxed{b \approx 41.0}$

$$\frac{\sin 92^\circ}{b} = \frac{\sin C}{29}$$

$A = 180^\circ - 92^\circ - 45^\circ$

$\boxed{A \approx 43.0^\circ}$

$$\sin^{-1}\left(\frac{29\sin 92^\circ}{b}\right) = C$$

$\boxed{C \approx 45.0^\circ}$

(27) $\boxed{\angle C = 19^\circ}$ Law of Sines

$$\frac{\sin 46^\circ}{31} = \frac{\sin 115^\circ}{b}$$

$\boxed{b \approx 39.1}$

$$\frac{\sin 115^\circ}{b} = \frac{\sin 19^\circ}{c}$$

$\boxed{c \approx 14.0}$

(28) $\angle A = 180^\circ - 68^\circ - 18^\circ = \boxed{94^\circ}$

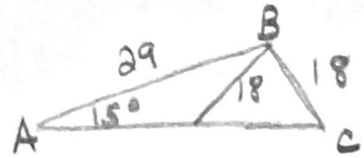
$$\frac{\sin 94^\circ}{29} = \frac{\sin 18^\circ}{c}$$

$\boxed{c \approx 9.0}$

$$\frac{\sin 94^\circ}{29} = \frac{\sin 68^\circ}{b}$$

$\boxed{b \approx 27.0}$

(29)

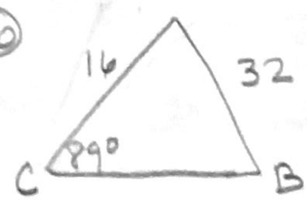


$$\frac{\sin 15^\circ}{18} = \frac{\sin C}{29}$$

$\boxed{\text{2 possible triangles}}$

$C \approx 24.6^\circ$ or 155.4°

(30)



$$\frac{\sin 89^\circ}{32} = \frac{\sin B}{16}$$

$B \approx 30.0^\circ$

$\boxed{\text{1 possible triangle}}$

(31) $2\sin\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}}$

(32) $3\cot\theta = -3$

$\cot\theta = -1$

$\tan\theta = -1$

$\boxed{\theta = \frac{3\pi}{4}, \frac{7\pi}{4}}$

(33) $4\cos\left(\theta + \frac{3\pi}{4}\right) = 2$

$\cos\left(\theta + \frac{3\pi}{4}\right) = \frac{1}{2}$

$\theta + \frac{3\pi}{4} = \frac{\pi}{3} \pm 2k\pi$ $\quad$ $\theta + \frac{3\pi}{4} = \frac{5\pi}{3} \pm 2k\pi$

$\theta = \frac{\pi}{3} - \frac{3\pi}{4} \pm 2k\pi$ $\quad$ $\theta = \frac{5\pi}{3} - \frac{3\pi}{4} \pm 2k\pi$

$\theta = \frac{4\pi}{12} - \frac{9\pi}{12} \pm \frac{24k\pi}{12}$ $\quad$ $\theta = \frac{20\pi}{12} - \frac{9\pi}{12} \pm 2k\pi$

$\theta = -\frac{5\pi}{12} \pm \frac{24k\pi}{12}$ $\quad$ $\theta = \frac{11\pi}{12} \pm \frac{24k\pi}{12}$

$\boxed{\theta = \frac{19\pi}{12}, \frac{11\pi}{12}}$

(34) $\tan\theta + \sqrt{2}\tan\theta\sin\theta = 0$

$\tan\theta(1+\sqrt{2}\sin\theta) = 0$

$\tan\theta = 0 \qquad 1+\sqrt{2}\sin\theta = 0$

$\frac{\sin\theta}{\cos\theta} = 0 \qquad \sin\theta = \frac{-1}{\sqrt{2}}$

$\boxed{\theta = 0, \pi} \qquad \sin\theta = -\frac{\sqrt{2}}{2}$

$\boxed{\theta = \frac{5\pi}{4}, \frac{7\pi}{4}}$

(35) $-2\cos^2\theta + \cos\theta + 2 = 1$

$2\cos^2\theta - \cos\theta - 2 = -1$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$2\cos\theta + 1 = 0 \qquad \cos\theta - 1 = 0$

$\cos\theta = -\frac{1}{2} \qquad \cos\theta = 1$

$\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}} \qquad \boxed{\theta = 0}$

(36) $4 = -\tan^2\theta + 7$

$\sqrt{\tan^2\theta} = \sqrt{3}$

$\tan\theta = \pm\sqrt{3}$

$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

(37) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$

(38) $\csc^{-1}(-\sqrt{2})$

$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\pi}{4}}$

(39) $\sec^{-1}\left(\tan\frac{\pi}{4}\right)$

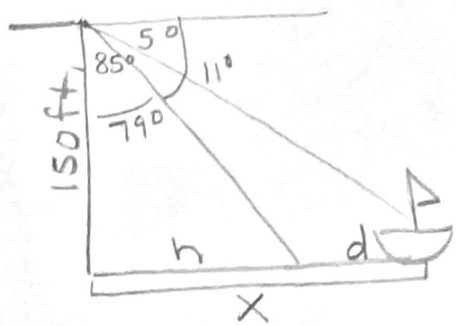
$\sec^{-1}(1) = \cos^{-1}(1) = \boxed{0}$

(40) $\tan^{-1}\left(\csc -\frac{\pi}{2}\right)$

$\tan^{-1}\left(\frac{1}{\sin -\frac{\pi}{2}}\right)$

$\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$

(41)



$\tan 85° = \frac{x}{150} \qquad \tan 79° = \frac{h}{150}$

$150\tan 85° = x \qquad 150\tan 79° = h$

$d = x - h$

$d = 150\tan 85° - 150\tan 79°$

$d \approx \boxed{942.8 \text{ ft.}}$

(42) $f = \langle -12, -5 \rangle \qquad \|f\| = 13$

$-5f = \boxed{\langle 60, 25 \rangle} \qquad \|-5f\| = \sqrt{60^2 + 25^2}$

$\|-5f\| = 65$

(43) $\overrightarrow{TX} = \langle -3 - -5, -10 - -1 \rangle$

$\boxed{\overrightarrow{TX} = \langle 2, -9 \rangle} \qquad \|\overrightarrow{TX}\| = \sqrt{2^2 + (-9)^2}$

$\boxed{\sqrt{2}\cdot\overrightarrow{TX} = \langle 2\sqrt{2}, -9\sqrt{2} \rangle} \qquad = \boxed{\sqrt{85}}$

$\|\sqrt{2}\cdot\overrightarrow{TX}\| = \sqrt{(2\sqrt{2})^2 + (-9\sqrt{2})^2}$

$= \sqrt{8 + 162}$

$= \boxed{\sqrt{170}}$

44 $\overrightarrow{TX} = \langle -10-1,\ 9--6 \rangle = \langle -11, 15 \rangle$

$\overrightarrow{YZ} = \langle -10-4,\ 8-2 \rangle = \langle -14, 6 \rangle$

$-\langle -11, 15 \rangle + \langle -14, 6 \rangle$

$\langle 11, -15 \rangle + \langle -14, 6 \rangle$

$\langle -3, -9 \rangle$

$\sqrt{(-3)^2 + (-9)^2} = \sqrt{90} = \boxed{3\sqrt{10}}$

45 $-f - g$

$-\langle -3, 10 \rangle - \langle 3, -6 \rangle$

$\langle 3, -10 \rangle + \langle -3, 6 \rangle$

$\boxed{\langle 0, -4 \rangle}$

$\downarrow 4$

$\sqrt{0^2 + (-4)^2}$

$\boxed{4}$

46 $u \cdot v$

$3 \cdot 1 + -3 \cdot -8$

$3 + 24$

$\boxed{27}$

47 $u \cdot v$

$-8 \cdot -8 + 5 \cdot 0$

$64 + 0$

$\boxed{64}$

48 $a = \langle -8, 1 \rangle$

$\|a\| \sqrt{(-8)^2 + 1^2}$

$\sqrt{65}$

unit vector $\left\langle \frac{-8}{\sqrt{65}}, \frac{1}{\sqrt{65}} \right\rangle$

or $\left\langle \frac{-8\sqrt{65}}{65}, \frac{\sqrt{65}}{65} \right\rangle$

49 $f = \langle -12, 16 \rangle$

$\|f\| = \sqrt{(-12)^2 + 16^2} = 20$

unit vector $\left\langle \frac{-12}{20}, \frac{16}{20} \right\rangle$

$\boxed{\left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle}$

50 $r = \frac{3}{5}$

recursive
$\boxed{\begin{cases} a_1 = \frac{6}{5} \\ a_n = \frac{3}{5} a_{n-1} \end{cases}}$

explicit
$\boxed{a_n = \frac{6}{5}\left(\frac{3}{5}\right)^{n-1}}$

51 $0, -2, -4, -6, \ldots$

$d = -2$

recursive
$a_1 = 0$
$a_n = a_{n-1} - 2$

explicit
$a_n = 0 - 2(n-1)$
$a_n = -2n + 2$

52 $a_1 = 4(1)^2 - 4 = 0$

$a_2 = 4(2)^2 - 4 = 12$

$a_3 = 4(3)^2 - 4 = 32$

$a_4 = 4(4)^2 - 4 = 60$

$a_5 = 4(5)^2 - 4 = 96$

$a_6 = 4(6)^2 - 4 = 140$

$a_7 = 4(7)^2 - 4 = 192$

$0 + 12 + 32 + 60 + 96 + 140 + 192$

$\boxed{532}$

55 $r = \frac{1}{3}$ converges to $S = \frac{1}{1 - \frac{1}{3}}$

$= \frac{1}{2/3} = \boxed{\frac{3}{2}}$

56 $r = -3$

diverges

$\therefore$ no sum

53 $\sum_{a=2}^{6} a + 300$ ← Linear so, arithmetic

$302 + 303 + 304 + 305 + 306$

$\boxed{1520}$

or $S = \frac{5}{2}(302 + 306)$

$= \boxed{1520}$

54 $r = -\frac{1}{5}$ $S = \frac{1}{1 + \frac{1}{5}} = \frac{1}{\frac{6}{5}} = \boxed{\frac{5}{6}}$

converges to

57 $r = \frac{1}{2}$

$S = \frac{1}{1 - 1/2} = \frac{1}{2/3} = \frac{3}{2}$

converges to

(58) $d = 5$

$S_n = \frac{n}{2}(a_1 + a_n)$

$986 = \frac{n}{2}(18 + a_n)$

$a_n = 18 + 5(n-1)$

$a_n = 18 + 5n - 5$

$a_n = 13 + 5n$

$986 = \frac{n}{2}(18 + 13 + 5n)$

$986 = \frac{n}{2}(31 + 5n)$

$0 = 2.5n^2 + 15.5n - 986$

$n = \boxed{17 \text{ terms}}$

(59) $r = 6$ $\quad S_n = -3110$

$S_n = \frac{-2(1-(-6)^n)}{1 - -6}$

$-3110 = \frac{\cancel{+}2(1-(-6)^n)}{\cancel{+}5}$

$-7775 = (1-(-6)^n)$

$-7776 = (-6)^n$

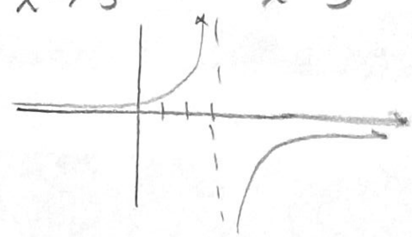
$\frac{\ln(7776)}{\ln(6)} = \frac{n \cancel{\ln(6)}}{\cancel{\ln(6)}}$

$n = 6$ $\quad \boxed{6 \text{ terms}}$

(60) $\sum_{n=1}^{5} 4^n$

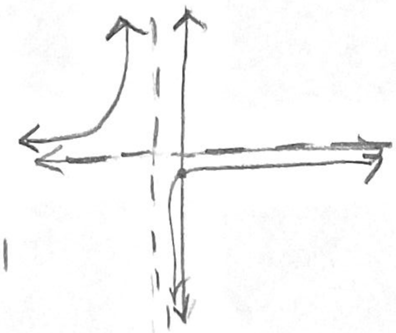
(61) $\sum_{n=1}^{6} 5 + 5n$

(62) $\lim_{x \to 3^-} -\frac{3x}{x-3} = \boxed{\infty}$

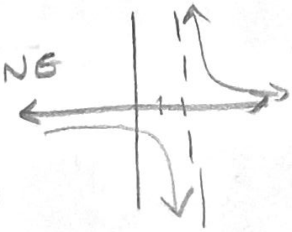


(63) $\lim_{x \to -1^+} \frac{-x+1}{x^2-1} = \boxed{-\infty}$

$\lim_{x \to -1^+} \frac{-\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = -\frac{1}{x+1}$



(64) $\lim_{x \to 2} \frac{1}{x-2} = \text{DNE}$



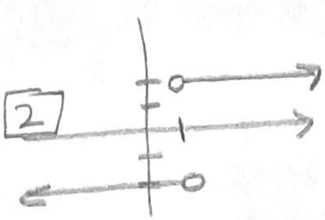
$\lim_{x \to 2^-} \frac{1}{x-2} = -\infty$

$\lim_{x \to 2^+} \frac{1}{x-2} = \infty$

(65) $\lim_{x \to -4} \frac{\cancel{(x+4)}}{(x+3)\cancel{(x+4)}}$

$\frac{1}{-4+3} = \boxed{-1}$

(66) $\lim_{x \to 1^+} \frac{2\cancel{(x-1)}}{\cancel{|x-1|}} = \boxed{2}$

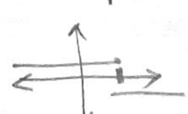


$\lim_{x \to 1^-} \frac{2(x-1)}{|x-1|} = -2$

(67) $-(-1)^2 - 6(-1) - 9$

$-1 + 6 - 9 = \boxed{-4}$

(68)



$\lim_{x \to 4} \frac{-x+4}{|-x+4|} = \boxed{\text{DNE}}$

(69) $\frac{2}{2} + \frac{3}{2}$ $(2, \frac{5}{2})$

$-2 + 6$ $(2, 4)$

$\boxed{\text{DNE}}$