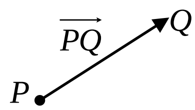
Pre-Calculus Name: ________________________

Notes—(6.1) Vectors in a Plane

- Quantities that have both ________________________ & ____________________ are represented by vectors.

- Vectors are defined by **magnitude** and **direction**. (NOT by **location**)



- Lowercase **boldface** letters such as **v, u** and **w** are used to represent vectors.

  or

- Two vectors are equal if their corresponding directed line segments have the same *length* & *direction*.

- Two vectors are equal if and only if they have the same ***component form***.

Component Form of Vector:

"Component form" means we have an initial point at (0,0) and terminal point (v_1, v_2)

- To find component form of a vector with initial point (x_1,y_1) and terminal point (x_2, y_2):

$$\langle \boldsymbol{v_1}, \boldsymbol{v_2} \rangle = \langle \boldsymbol{x_2} - \boldsymbol{x_1}, \boldsymbol{y_2} - \boldsymbol{y_1} \rangle$$

- v_1 is the horizontal component

$$\boldsymbol{v_1} = \boldsymbol{x_2} - \boldsymbol{x_1}$$

- $\boldsymbol{v_2}$ is the vertical component

$$\boldsymbol{v_2} = \boldsymbol{y_2} - \boldsymbol{y_1}$$

Ex 1) Let ***u*** be the vector represented by the directed line segment from *R* = (–4, 2) to *S* = (–1, 6), & ***v*** the vector from *O* (0, 0) to *P* = (3, 4). Prove that ***u*** = ***v.***

Ex 2) Let ***u*** be the vector represented by the directed line segment from *R* = (7, –3) to *S* = (4, –5), & ***v*** the vector from *O* (0, 0) to *P* = (–3, –2). Prove ***u*** = ***v***

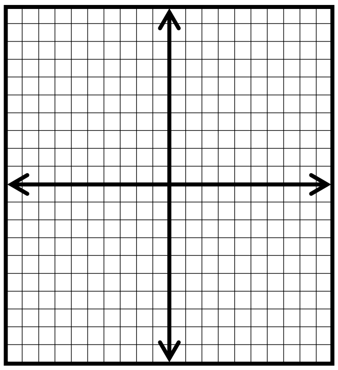
The ***magnitude*** (or length) of vector ***v*** = $\overrightarrow{PQ}$ determined by $P = (x_1, y_1)$ and $Q = (x_2, y_2)$

$$\|\mathbf{v}\| = \sqrt{{v_1}^2 + {v_2}^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

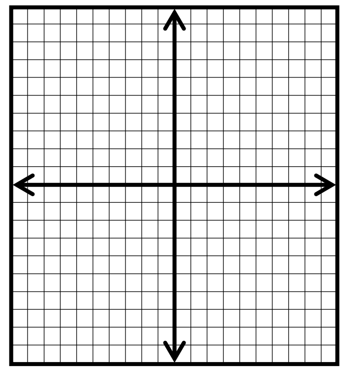
Note: *The vector* $\mathbf{0} = \langle 0,0\rangle$, *called the zero vector, has 0 length and 0 direction.*
Practice:

Ex 3) $P=(-3, 1)$ *and* $Q = (-6, 5)$ Find the component form & magnitude of vector

a) $\overrightarrow{PQ}$



b) $\overrightarrow{QP}$



DEFINITION --- Vector Addition and Scalar Multiplication

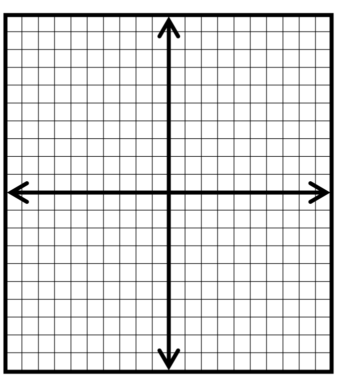
Let ***u*** = $\langle u_1, u_2\rangle$ and ***v*** = $\langle v_1, v_2\rangle$be vectors and let *k* be a real number (scalar).

The **sum** (or ***resultant vector***) of ***u*** + ***v*** is the vector: ***u*** + ***v*** **=** $\langle u_1 + v_1, u_2 + v_1\rangle$

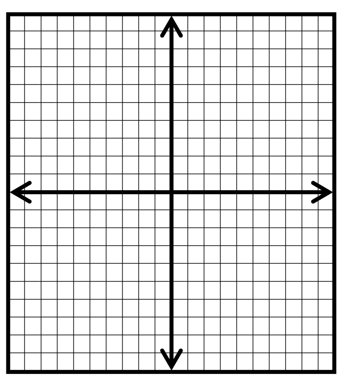
The **scalar product** of vector ***u*** and scalar *k* is the vector: k**u** = $\langle ku_1, ku_2\rangle$

Ex 4) Given ***u*** = $\langle -1, 2\rangle$ and ***v*** = $\langle 2, 5\rangle$find the component form each of the following vectors:

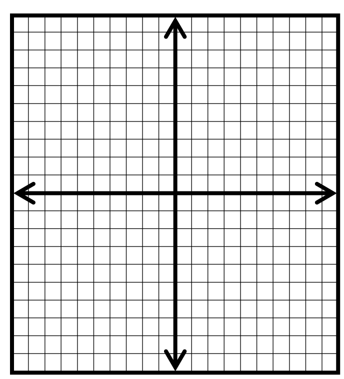
a) 3***u***



b)* *u + ***v***



c) *3**u** –**v***



Now You Try ☺

Ex 5) Given ***u*** = $\langle 5, -2\rangle$ and ***v*** = $\langle 6, 4\rangle$find the component form each of the following vectors:

a) **u** – ***v*** b) 5**u** c) 3**u** + (–2)***v***

DEFINITION --- Unit Vectors and the standard Unit Vectors

A vector ***u*** with length 1 is called a ______________ ______________. to create a unit vector ***u*** in the direction of ***v*** simply divide vector ***v*** by its magnitude: $\mathbf{u} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{\mathbf{1}}{|\mathbf{v}|}\mathbf{v}$

The two unit vectors ***i*** = ⟨1, 0⟩ and ***j* =** ⟨0, 1⟩ are the **standard unit vectors** and can be used to write a vector as a linear combination of ***i*** & ***j***.

Ex5) Find a unit vector in the direction of ***v*** = ⟨–3, 2⟩, and verify that it has a length equal to 1. Then write the answer in both component form and as a linear combination of the standard unit vectors.

|*v*| = ____________

The component form of the vector **u**
a unit vector in the direction of **v** is __________

u written as a linear combination
of the standard unit vectors ***i*** & ***j*** is __________

DEFINITION --- Direction Angle

To precisely specify the direction of a vector state its **direction angle** θ (made by the vector and the positive *x*-axis)

Using trigonometry, we can see the horizontal component of a vector ***v*** is (|***v***|cos θ) and the vertical component
is (|***v***|sin θ), thus: ***v*** = (|***v***|cos θ)***i*** + (|***v***|sin θ)***j*** = ⟨|***v***|cos θ, |***v***|sin θ⟩

Ex6) Find the components of vector ***v*** with direction angle $\theta = 115^\circ$ and magnitude of 6.

Ex7) Find the magnitude & direction angle of each vector:

a) ***u*** = ⟨3, 2⟩ **b)** ***v*** = ⟨–2, –5⟩