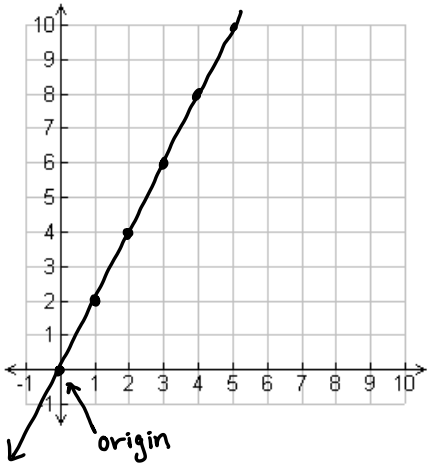
Direct Variation

$y = 2x$

x	y	$\frac{y}{x}$
0	0	
1	2	2
2	4	2
3	6	2
4	8	2
5	10	2



y varies directly as x.

$y = kx$

↑ Constant of variation

$\frac{y}{x} = k$

- A **direct variation** is described as a **linear equation** in the form $y = kx$. You have a direct variation when your x and y values increase or decrease together at the same rate.

- k represents the **constant of variation**. This describes how quickly your values are increasing or decreasing. To find your constant of variation, simply divide $\left(k = \frac{y}{x}\right)$

- A **direct variation** only exists if $\frac{y}{x} = \frac{y}{x} = \frac{y}{x}$ for every set of points in your table.

Example 1 For the data in the table below, determine if you have a direct variation. If yes, find the **constant of variation**. Use the constant of variation to write an equation.

Hrs. Worked	Total Pay	$\frac{y}{x}$
2	15	$=7.50$
3	22.50	$=7.50$
4	30	
5	37.50	

$y = 7.5x$

Gallons of Gas ↑	Total Cost ↑
5	31.25
8	34.48
11	36.72
13	39.78

Ex 2: If y varies directly as x, and y is 6 when x is 9, what is x when y is 12?

① $y = kx$

$\frac{6}{9} = \frac{k(9)}{9}$

$\frac{2}{3} = k$

② Find k

Eqn: $\boxed{y = \frac{2}{3}x}$

$\frac{3}{2}\cancel{12}^{6} = \frac{3}{2}\frac{2}{3}x$

$18 = x$

Ex 3: The distance a spring stretches $\boxed{\text{varies}}$ directly with the amount of weight that is hanging on it. A weight of 2.5 pounds stretches a spring 18 inches. Find the stretch of a spring when a weight of 6.4 pounds is hanging on it.

① $d = kw$

$\frac{18}{2.5} = \frac{k(2.5)}{2.5}$

$7.2 = k$

② Find k

$\boxed{d = 7.2w}$

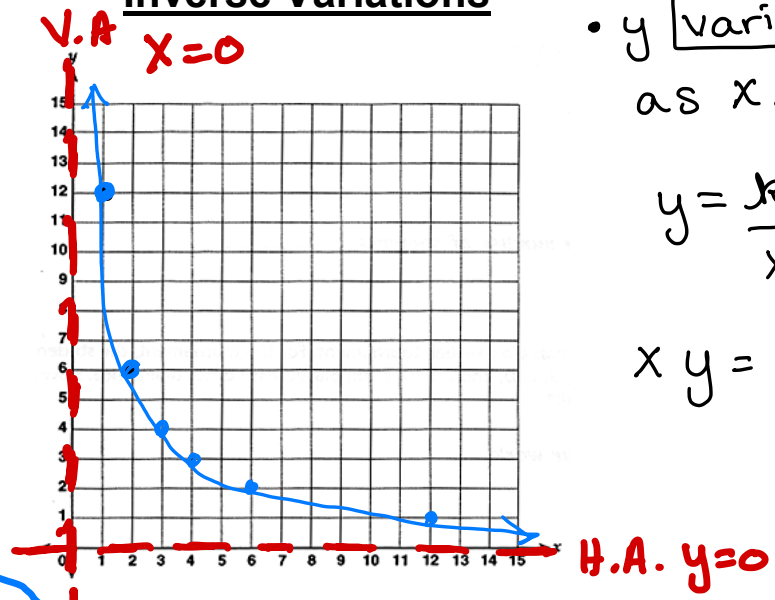
$d = 7.2(6.4)$

$d = 46.08$ inches

Inverse Variations

constant

l	w	A
x	y	xy
1	12	12
2	6	12
3	4	12
4	3	12
6	2	12



- y $\boxed{\text{varies}}$ inversely as x.

$$y = \frac{k}{x}$$

$$xy = k$$

- An **inverse variation** occurs when one value increases while the other decreases. An inverse variation can be described using the equation $y = \frac{k}{x}$

- $\boldsymbol{k}$ represents the **constant of variation**. To find your constant of variation, simply multiply $(k = xy)$

- An inverse variation only exists if $xy = xy = xy$ for every set of points in your table.

Ex 1: The table below shows 3 different methods of traveling across from NY to LA. One person flies, another drives, and one person rides a bike! Determine whether you have an inverse variation. If yes, use the constant of variation to write an equation to represent your data.

	MPH	# hours to cover 3000 miles	
flies	500	6	= 3000
drive	60	50	= 3000
bike	15	200	= 3000

Ex 3: If y varies inversely with x, and y = 25 when x = 3, what is x when y is 85?

$y = \frac{k}{x}$ Eqn. $y = \frac{75}{x}$ $x\,85 = \frac{75}{\cancel{x}}\cancel{x}$ $x = \frac{75}{85}$ $x = \frac{15}{17}$

Ex 4: If y varies inversely with x, and y = 10 when x = 15, what is y when x is 30?

① $y = \frac{k}{x}$ $10 = \frac{k}{15}$ $150 = k$ $y = \frac{150}{x}$ $y = \frac{150}{30}$ $y = 5$ $y = 5$

Ex 5: The number of hours it takes for a block of ice to melt $\boxed{\text{varies}}$ inversely as the temperature If it takes 2 hours for a square inch of ice to melt at 65º, how long does it take for a block of ice to melt at 40 º?

$h = \frac{k}{T}$ $2 = \frac{k}{65}$ $130 = k$ $h = \frac{130}{T}$ $h = \frac{130}{40}$ $h = 3.25$ hours

Joint Variations

- A **joint variation** is the same as direct variation, but with 2 or more quantities.
- An **equation** for a joint variation is in the form $z = kxy$
- If z **varies jointly** with x and y, we say $\frac{z}{xy} = \frac{z}{xy}$. The **constant of variation** is $k = \frac{z}{xy}$.

Ex 1: Assume *z* varies jointly with *x* and *y*. Given that *z* = 12 when *x* = 1 and *y* = 6. If *x* = 2 and *y* = 3, find the value of *z*.

$z = kxy$

$12 = k(1)(6)$

$k = 2$

$z = 2xy$

$z = 2(2)(3)$

$z = 12$

Ex 2: The temperature, T, of an enclosed gas varies jointly with the product of the volume, V and the pressure, P. The temperature of a gas is 294° when the volume is 8000 cubic centimeters and the pressure is 0.75 kilogram per square centimeter. What is the temperature when the volume is 7000 cubic centimeters and the pressure is 0.87 kilogram per square centimeter?

$T = kVP$

$\frac{294}{8000(.75)} = \frac{k(8000)(.75)}{8000(.75)}$

$k = .049$

$T = .049VP$

$T = .049(7000)(.87)$

$T = 298.41°$

Ex 3: The energy, e, used when lifting an object varies jointly to the product of the mass, m, of the object and the height, h, that the object is lifted. An object with a mass of 120-kilogram lifted 1.8 meters above the ground requires 2116.8 joules of energy. How much energy is needed when lifting an object with a mass of 100 kg to a height 1.5 meters off the ground?

$e = kmh$

$\frac{2116.8}{120(1.8)} = \frac{k(120)(1.8)}{(120)(1.8)}$

$e = 9.8mh$

$e = 9.8(100)(1.5)$

$e = 1470$ joules

Combined Variations

Ex 4: z varies directly with the square of x and inversely with y. If x = 2, y = 4, and z = 3, find z when x = 4 and y = 9.

$z = \frac{kx^2}{y}$

$3 = \frac{k(2)^2}{4}$

$k = 3$

$z = \frac{3x^2}{y}$

$z = \frac{3(4)^2}{9}$

$z = \frac{16}{3}$

Ex 5: In building a brick wall, the amount of time it takes to complete the wall varies directly with the number of bricks in the wall and varies inversely with the number of workers that are working together. A wall containing 1200 bricks, using 3 workers, takes 18 hours to build. How long would it take to build a wall of 4500 bricks if 5 people worked on it?

$t = \frac{kb}{w}$

$18 = \frac{k(1200)}{3}$

$\frac{18}{400} = \frac{k(400)}{400}$

$k = .045$

$k = \frac{9}{200}$

$t = \frac{.045b}{w}$

$t = \frac{9b}{200w}$

$t = \frac{.045(4500)}{5}$

$t = 40.5$ hours

Ex 6: The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. A wire with a length of 200 inches and a diameter of one-quarter of an inch has a resistance of 20 ohms. Find the electrical resistance in a 500 inch wire with the same diameter.

$R = \frac{kl}{d^2}$

$20 = \frac{k(200)}{(.25)^2}$

$R = \frac{.00625l}{d^2}$

$R = \frac{.00625(500)}{(.25)^2}$

$R = 50$ ohms