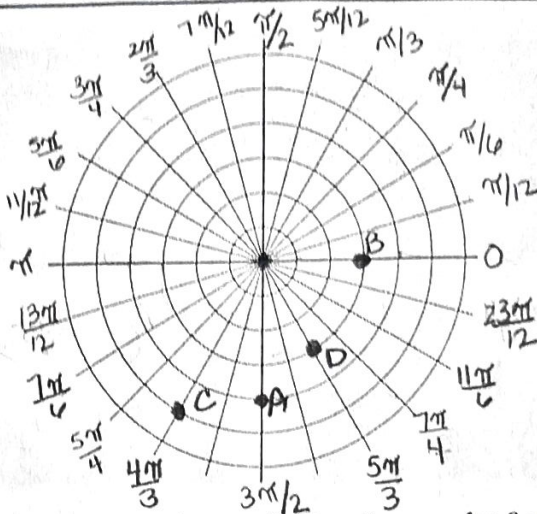


Precalculus

Polar Review

Plot each polar coordinate on the graph.

1. $A(4, 270^\circ)$
2. $B(-3, \pi)$
3. $C(5, \frac{4\pi}{3})$
4. $D(-3, 120^\circ)$



Convert each of the following rectangular coordinates to polar coordinates. Answers for θ should be in radians, exact answers if possible. If you need to have decimals, please round to the nearest thousandth.

5. $(-3, 3)$

$(3\sqrt{2}, \frac{3\pi}{4})$ or $(3\sqrt{2}, \frac{5\pi}{4})$
 or $(-3\sqrt{2}, -\frac{\pi}{4})$ or $(-3\sqrt{2}, \frac{7\pi}{4})$

6. $(-2, -2\sqrt{3})$

$\tan \theta = \frac{-2\sqrt{3}}{-2}$
 $\theta = \tan^{-1}(\sqrt{3})$
 $\theta = \frac{\pi}{3}$

$(4, \frac{\pi}{3})$ or $(4, \frac{4\pi}{3})$

7. $(-0.8, -2.1)$

$(2.247, 4.348)$
 or $(-2.247, 1.2)$

$\tan \theta = \frac{-2.1}{-0.8} \approx 2.625$
 $\theta \approx 1.2$
 $(-0.8)^2 + (-2.1)^2 = c^2$
 $c \approx 2.247$

8. $(-2.3, 0.2)$

$(2)^2 + (-2.3)^2 = c^2$
 $c \approx 2.3$

$(-2.3, -0.0867)$
 or $(2.3, 3.055)$

$\tan \theta = \frac{0.2}{-2.3}$
 $\theta = -0.0867$

Convert each of the following polar coordinates to rectangular coordinates. Decimals should be rounded to the nearest thousandth.

9. $(5, 300^\circ)$

$(5 \cos 300^\circ, 5 \sin 300^\circ)$
 $(\frac{5}{2}, -\frac{5\sqrt{3}}{2})$

10. $(4, \frac{3\pi}{2})$

$(4 \cos \frac{3\pi}{2}, 4 \sin(\frac{3\pi}{2}))$
 $(0, -4)$

11. $(-3.1, 182^\circ)$

$(-3.1 \cos 182^\circ, -3.1 \sin 182^\circ)$
 $(3.098, 0.108)$

12. $(8.1, 5.2)$

$(8.1 \cos 5.2, 8.1 \sin 5.2)$
 $(3.795, -7.156)$

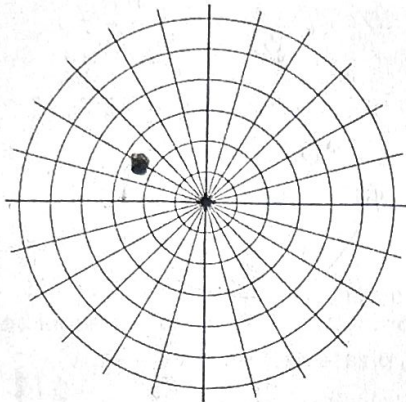
Find the other 3 polar coordinates that represent the same point for $-2\pi \leq \theta \leq 2\pi$.

13. $(4, \frac{3\pi}{4})$ $(-4, -\frac{\pi}{4})$ $(4, \frac{5\pi}{4})$
 $(-4, \frac{7\pi}{4})$

14. $(-2, -\frac{2\pi}{3})$

$(2, \frac{\pi}{3})$
 $(2, \frac{5\pi}{3})$
 $(-2, \frac{4\pi}{3})$

15. Find all polar coordinates for the point such that $-2\pi \leq \theta \leq 2\pi$.



$(2.5, \frac{5\pi}{6})$
 $(-2.5, -\frac{\pi}{6})$
 $(2.5, -\frac{7\pi}{6})$
 $(-2.5, \frac{11\pi}{6})$

Find the other 3 polar coordinates that represent the same point for $-360^\circ \leq \theta \leq 360^\circ$.

16. $(3, -20^\circ)$

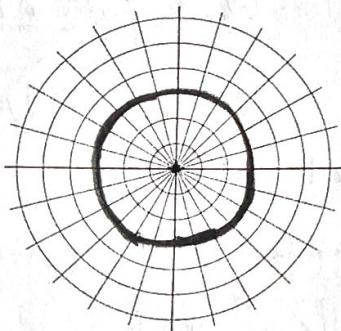
$(3, 340^\circ)$
 $(-3, 160^\circ)$
 $(-3, -200^\circ)$

17. $(-4, 103^\circ)$

$(4, -77^\circ)$ $(-4, -257^\circ)$
 $(4, 283^\circ)$

Graph each polar equation. Then convert the equation to rectangular form.

18. $r = 3$

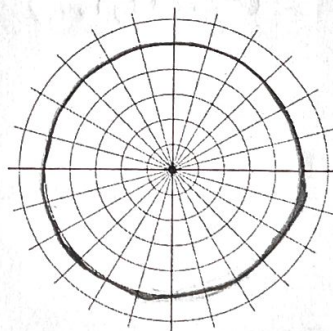


rectangular equation:

$r^2 = 9$

$x^2 + y^2 = 9$

19. $r = 5$

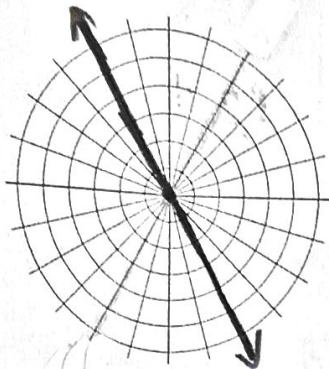


rectangular equation:

$r^2 = 25$

$x^2 + y^2 = 25$

20. $\theta = \frac{2\pi}{3}$



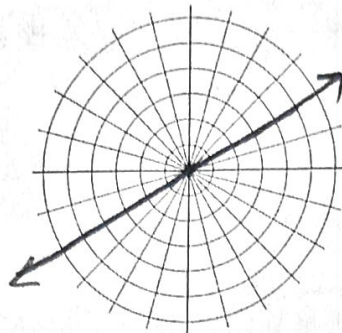
rectangular equation:

$$\tan \theta = \tan \frac{2\pi}{3}$$

$$\frac{y}{x} = -\sqrt{3}$$

$$y = -\sqrt{3}x$$

21. $\theta = 30^\circ$



rectangular equation:

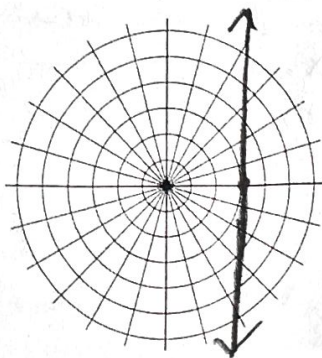
$$\tan \theta = \tan 30^\circ$$

$$\frac{y}{x} = \frac{1}{\sqrt{3}}$$

$$y = \frac{\sqrt{3}}{3}x$$

Convert the following polar equations into rectangular equations so that you can graph them.

22. $r = 3 \sec \theta$



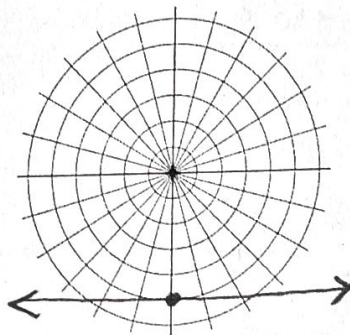
$$r = \frac{3}{\cos \theta}$$

$$r \cos \theta = 3$$

$$x = 3$$

Vertical line

23. $r = -5 \csc \theta$



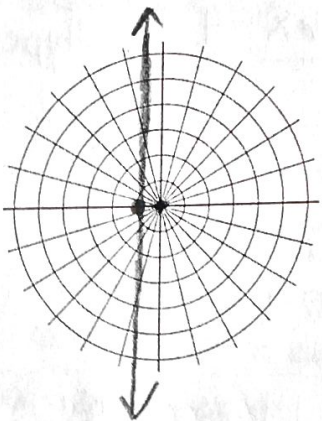
$$r = \frac{-5}{\sin \theta}$$

$$r \sin \theta = -5$$

$$y = -5$$

horizontal line

24. $r = -\frac{1}{\cos \theta}$

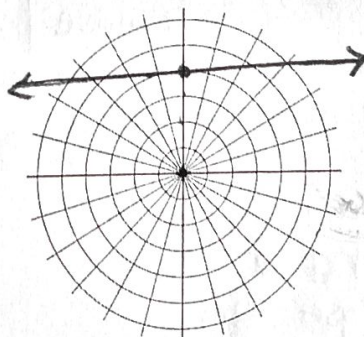


$$r \cos \theta = -1$$

$$x = -1$$

Vertical line

25. $r = \frac{4}{\sin \theta}$



$$r \sin \theta = 4$$

$$y = 4$$

horizontal line

26. $r = 4 \sin \theta$

$r^2 = 4r \sin \theta$

$x^2 + y^2 = 4y$

$x^2 + y^2 - 4y = 0$ Circle

28. $r = 5 \cos \theta$

$r^2 = 5r \cos \theta$

$x^2 + y^2 = 5x$

$x^2 + y^2 - 5x = 0$ Circle

27. $r = -6 \cos \theta$

$r^2 = -6r \cos \theta$

$x^2 + y^2 = -6x$

$x^2 + y^2 + 6x = 0$ Circle

29. $r = -2 \sin \theta$

$r^2 = -2r \sin \theta$

$x^2 + y^2 = -2y$

$x^2 + y^2 + 2y = 0$ Circle

Convert each of the following polar equations to rectangular form (standard conic form). Classify the conic.

30. $r = 2 \sin \theta - 6 \cos \theta$

$r^2 = 2r \sin \theta - 6r \cos \theta$

$x^2 + y^2 = 2y - 6x$

$x^2 + y^2 + 6x - 2y = 0$

Circle

31. $r = \frac{1}{1 + \cos \theta}$

$r + r \cos \theta = 1$

$r + x = 1$

$(r)^2 = (1-x)^2$

$x^2 + y^2 = 1 - 2x + x^2$

$y^2 - 1 = -2x$

33. $r = \frac{2}{1 + 4 \cos \theta}$

$r + 4r \cos \theta = 2$

$r + 4x = 2$

$r^2 = (2 - 4x)^2$

$x^2 + y^2 = 4 - 16x + 16x^2$

$y^2 - 15x^2 + 16x - 4 = 0$ Hyperbola

$x = -\frac{1}{2}y^2 + \frac{1}{2}$

Parabola

32. $r = \frac{3}{5 - 2 \sin \theta}$

$5r - 2r \sin \theta = 3$

$5r - 2y = 3$

$(5r)^2 = (2y + 3)^2$

$25r^2 = 4y^2 + 12y + 9$

$25x^2 + 25y^2 = 4y^2 + 12y + 9$

$25x^2 + 21y^2 - 12y - 9 = 0$ Ellipse

Convert each of the following rectangular equations to polar form.

34. $\frac{y^2}{4} = x$

$x^2 y^2 = 4x^2 + x^2 + 4 - 4$

$r^2 \sin^2 \theta = 4r \cos \theta$

$r^2 \sin^2 \theta - 4r \cos \theta = 0$

$r(r \sin^2 \theta - 4 \cos \theta) = 0$

$r = \frac{4 \cos \theta}{\sin^2 \theta}$

36. $y = -x$

$\frac{y}{x} = -1$

$\tan \theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = \frac{3\pi}{4}$ or $\theta = -\frac{\pi}{4}$

35. $(x - 7)^2 + (y + 2)^2 = 53$

$x^2 - 14x + 49 + y^2 + 4y + 4 = 53$

$r^2 - 14r \cos \theta + 4r \sin \theta = 0$

$r - 14 \cos \theta + 4 \sin \theta = 0$

$r = 14 \cos \theta - 4 \sin \theta$

37. $y = \frac{\sqrt{3}}{3}x$

$\frac{y}{x} = \frac{1}{\sqrt{3}}$

$\tan \theta = \frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = \frac{\pi}{6}$

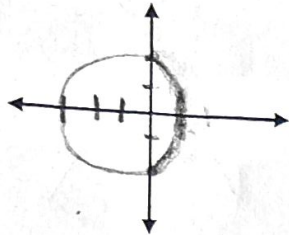
or $\theta = \frac{7\pi}{6}$



Identify and sketch a graph for the following.

38. $r = 2 - \cos \theta$

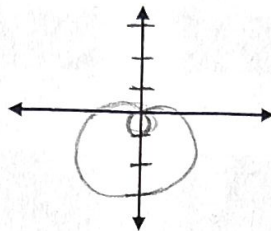
identification: Convex Limacon w/ loop



θ	r
π	1.1
$\frac{3\pi}{2}$	1.5
2π	2

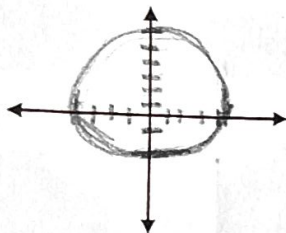
39. $r = 1 - 2 \sin \theta$

identification: Limacon w/ inner loop



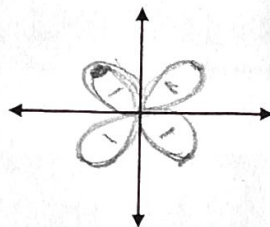
40. $r = 4 + 2 \sin \theta$

identification: Convex limaçon



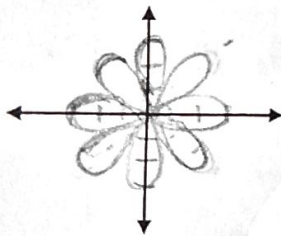
41. $r = 2 \sin 2\theta$

identification: rose w/ 4 petals



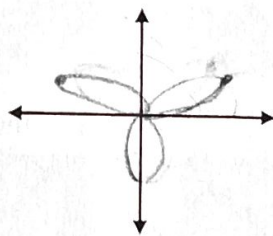
42. $r = 3 \cos 4\theta$

identification: Rose w 8 petals



43. $r = 6 \sin 3\theta$

identification: Rose w/ 3 petals



θ	r
$\frac{\pi}{3}$	2
$\frac{2\pi}{3}$	0
π	-6
$\frac{4\pi}{3}$	0
$\frac{3\pi}{2}$	0

θ	r
$\frac{\pi}{3}$	2
$\frac{2\pi}{3}$	0
π	-6
$\frac{4\pi}{3}$	0
$\frac{3\pi}{2}$	0

$\frac{2\pi}{3}$