

5.3 & 5.4

$$\begin{aligned} \textcircled{1} \quad \cot\theta \cdot \sec\theta &= \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} \\ &= \frac{1}{\sin\theta} \\ &= \boxed{\csc\theta} \quad \textcircled{B} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \frac{\sec\theta}{\csc\theta} &= \sec\theta \div \csc\theta \\ &= \frac{1}{\cos\theta} \div \frac{1}{\sin\theta} \\ &= \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} \\ &= \boxed{\frac{\sin\theta}{\cos\theta}} \quad \textcircled{C} \\ &= \tan\theta \end{aligned}$$

$$\textcircled{4} \quad (1+\cos x)(1-\cos x) \quad \textcircled{5} \quad \cos^2(4\theta) + \sin^2(4\theta) = \boxed{1} \quad \textcircled{A}$$

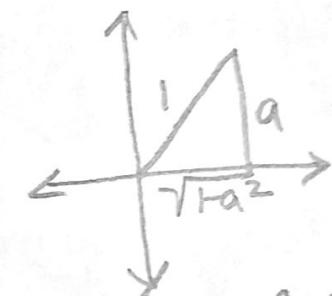
$$\begin{aligned} &= 1 - \cos^2 x \\ &= \boxed{\sin^2 x} \quad \textcircled{B} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad \sin A + \cot^2 A \sin A &= \sin A (1 + \cot^2 A) \\ &= \sin A \csc^2 A \end{aligned}$$

$$= \sin A \cdot \frac{1}{\sin^2 A}$$

$$= \frac{1}{\sin A}$$

$$= \boxed{\csc A} \quad \textcircled{D}$$



$$\begin{aligned} a^2 + b^2 &= 1^2 \\ b^2 &= 1 - a^2 \\ b &= \sqrt{1 - a^2} \end{aligned}$$

$$\cos\theta = \boxed{\sqrt{1 - a^2}} \quad \textcircled{C}$$

$$\textcircled{8} \quad \frac{\sin^2 B \cot B}{\cos B}$$

$$\begin{aligned} &= \sin^2 B \cdot \frac{\cos B}{\sin B} \cdot \frac{1}{\cos B} \\ &= \boxed{\sin B} \end{aligned}$$

$$\begin{aligned} 9) & \sin x - \sin x \cos^2 x \\ & \sin x (1 - \cos^2 x) \\ & \sin x (\sin^2 x) \\ & \boxed{\sin^3 x} \end{aligned}$$

$$10) \sin^2 x + \cos\left(\frac{\pi}{2} - x\right) - 1 + \cos^2 x$$

$\cancel{x} + \sin x \rightarrow \cancel{x}$

$\boxed{\sin x}$

Solve each of the following on the interval from $[0, 2\pi)$

$$\begin{aligned} 11) & 2\cos^2 x - 5\cos x + 3 = 0 \\ & (2\cos x + 1)(\cos x - 3) = 0 \\ & \cos x = -\frac{1}{2} \quad \cos x = 3 \\ & X = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \emptyset \end{aligned}$$

$$\begin{aligned} 13) & 4\cos^2 x - 3 = 0 \\ & \sqrt{\cos^2 x} = \sqrt{\frac{3}{4}} \\ & \cos x = \pm \frac{\sqrt{3}}{2} \end{aligned}$$

$$X = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Verify that each of the following is an identity

$$15) \frac{\cos^2 \theta}{\sin^2 \theta} + \csc \theta \sin \theta = \csc^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Q.E.D. ☺

$$\begin{aligned} 12) & \cos^2 x + 4\sin x + 4 = 0 \\ & 1 - \sin^2 x + 4\sin x + 4 = 0 \\ & -\sin^2 x + 4\sin x + 5 = 0 \\ & \sin^2 x - 4\sin x - 5 = 0 \\ & (\sin x + 1)(\sin x - 5) = 0 \\ & \sin x = -1 \quad \sin x = 5 \rightarrow \emptyset \end{aligned}$$

$$14) 2 \tan x \sin x + \tan x = 0 \rightarrow X = \frac{3\pi}{2}$$

$$\tan x (2\sin x + 1) = 0$$

$$\begin{aligned} \tan x = 0 & \quad 2\sin x + 1 = 0 \\ & \sin x = -\frac{1}{2} \\ X = 0, \pi & \quad X = \frac{7\pi}{6}, \frac{11\pi}{6} \end{aligned}$$

$$16) \frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = 2\sec^2 x$$

$$\begin{aligned} & \frac{1 + \sin x + 1 - \sin x}{(1 - \sin x)(1 + \sin x)} = \\ & \frac{2}{1 - \sin^2 x} = \\ & \frac{2}{\cos^2 x} = 2\sec^2 x \quad \checkmark \quad Q.E.D. \end{aligned}$$

$$17) \frac{1 + \cos(2\alpha)}{\sin(2\alpha)} = \cot \alpha$$

$$\begin{aligned} &= \frac{1 + 1 - 2\sin^2 \alpha}{2\sin \alpha \cos \alpha} \\ &= \frac{2 - 2\sin^2 \alpha}{2\sin \alpha \cos \alpha} \\ &= \frac{2(1 - \sin^2 \alpha)}{2\sin \alpha \cos \alpha} \\ &= \frac{\cos^2 \alpha}{\sin \alpha \cos \alpha} \end{aligned}$$

$$\begin{aligned} &= \frac{\cos \alpha}{\sin \alpha} = \cot \alpha \quad \checkmark \\ &= \frac{\cos \alpha}{\sin \alpha} = \cot \alpha \quad \text{Q.E.D.} \end{aligned}$$

$$18) \frac{1 + \cos(2\alpha)}{\sin(2\alpha)} = \cot \alpha$$

OOPS ☹