Key

Precalculus Unit 3 Review

1) There are two options that Caleb must consider for investing his $1200. One option is to put his money into an account that earns interest at 4% compounded quarterly for 2 years. The other option is to put his money into an account that earns interest at 12% compounded monthly for 2 years. Which option gives him the most money at the end of the 2-year span?

$A = 1200\left(1+\frac{.04}{4}\right)^{4(2)}$ $\boxed{A = \$1299.42}$

$A = 1200\left(1+\frac{.12}{12}\right)^{12(2)}$ * $\boxed{A = \$1523.68}$

2nd option

2) An isotope of cesium-137 has a half-life of 25 years. How much cesium-137 would remain from Timothy's 10 gram sample after 90 years? Round to the nearest hundredth.

$A = 10\left(\frac{1}{2}\right)^{\frac{90}{25}}$

$A = \boxed{.82\,g}$

3) A rumor spreads through a track team according to the model $R(t) = 162(1 - 3^{-t})$, where t is the number of hours since the rumor was started and R(t) is the number of people who have heard the rumor. How many hours will it take for 160 people to hear the rumor?

$\frac{160}{162} = \frac{162(1-3^{-t})}{162}$

$\frac{80}{81} = 1 - 3^{-t}$

$\frac{80}{81} - 1 = -3^{-t}$

$\frac{-1}{81} = -3^{-t}$

$3^{-4} = 3^{-t}$ $t = 4$ hours

4) You invest $2000 into an account that earns 3.2% interest compounded continuously. How long will it take for you to have $6500?

$\frac{6500}{2000} = \frac{\cancel{2000}\,e^{.032(t)}}{\cancel{2000}}$

$3.25 = e^{.032t}$

$\ln 3.25 = \ln e^{.032t}$

$\frac{\ln 3.25}{.032} = \frac{\cancel{.032}t}{\cancel{.032}}$

$t \approx \boxed{36.8 \text{ years}}$

Solve each equation. Write the exact answer and then write the decimal approximation.

5) $4^{2x-1} = \left(\frac{1}{16}\right)^{x-1} \cdot 64^{3x}$

$4^{2x-1} = (4^{-2})^{x-1} \cdot (4^3)^{3x}$

$4^{2x-1} = 4^{-2(x-1)+9x}$

$2x-1 = -2(x-1) + 9x$

$2x-1 = -2x+2+9x$

$2x-1 = 7x+2$

$-5x = 3$

$\boxed{x = -\frac{3}{5}}$

6) $\log_4 x + \log_4(x+6) = 2$

$\log_4[x(x+6)] = 2$

$4^2 = x^2 + 6x$

$16 = x^2 + 6x$

$0 = x^2 + 6x - 16$

$0 = (x+8)(x-2)$

$x = \cancel{-8}$ $\boxed{x = 2}$

7) $\log_2(2x-5)-\log_2(x-7)=\log_2 8$

$\log_2 \frac{2x-5}{x-7} = \log_2 8$

$\frac{2x-5}{x-7} = 8$

$2x-5 = 8(x-7)$

$2x-5 = 8x-56$

$\frac{51}{6} = \frac{6x}{6}$

$\boxed{8.5 = x}$

8) $2e^{2x+4}+5=26$

$2e^{2x+4} = 21$

$e^{2x+4} = 10.5$

$\ln 10.5 = 2x+4$

$\ln(10.5)-4 = 2x$

$x = \frac{1}{2}(\ln(10.5)-4)$

$\boxed{x \approx -.824}$

9) $4^{2x-1}=7^{3x+1}$

$(2x-1)\ln 4 = (3x+1)\ln 7$

$2x\ln 4 - \ln 4 = 3x\ln 7 + \ln 7$

$2x\ln 4 - 3x\ln 7 = \ln 4 + \ln 7$

$x(2\ln 4 - 3\ln 7) = \ln 4 + \ln 7$

$\boxed{x = \frac{\ln 4 + \ln 7}{2\ln 4 - 3\ln 7} \approx -1.087}$

10) $4(3)^{x+1}+15=3$

$4(3)^{x+1} = -12$

$3^{x+1} = -3$

not possible

$\boxed{\text{No Solution}}$

11) $2e^{2x}+12e^{x}=110$

$2e^{2x}+12e^{x} = 110$

$e^{2x}+6e^{x}-55 = 0$

$(e^{x}+11)(e^{x}-5) = 0$

$e^{x}+11=0 \quad e^{x}-5=0$

$e^{x}=-11 \quad e^{x}=5$

not possible $\quad \boxed{x=\ln 5}$

12) $\frac{243^{-2n}}{27} = 9^{2-n}$

$\frac{(3^5)^{-2n}}{3^3} = (3^2)^{2-n}$

$3^{-10n-3} = 3^{4-2n}$

$-10n-3 = 4-2n$

$-8n = 7$

$\boxed{n = -\frac{7}{8}}$

13) Find a logistic equation in the form $y = \frac{c}{1+ab^x}$ that fits the graph if the y-intercept is 5, ~~and~~ the point (24, 135) is on the curve, and a limit to growth of 150.

$y = \frac{150}{1+29(b)^x}$

$135 = \frac{150}{1+29(b)^{24}}$

$1+29(b)^{24} = \frac{150}{135}$

$1+29(b)^{24} = \frac{10}{9}$

$29(b)^{24} = \frac{1}{9}$

$b^{24} = \frac{1}{9\cdot 29}$

$b^{24} = \frac{1}{261}$

$b = \frac{1}{261}^{\frac{1}{24}}$

$b = 261^{-\frac{1}{24}}$

$\boxed{y = \frac{150}{1+29(\frac{1}{261})^{x/24}}}$

or

$\boxed{y = \frac{150}{1+29(261)^{-\frac{1}{24}x}}}$

14) The number of students infected with the flu after t days at Springfield High School is modeled by the function $P(t) = \frac{1600}{1+99e^{-.4t}}$.

a. What was the initial number of infected students? 16

b. After 5 days, how many students will be infected?

$P(5) = \frac{1600}{1+99e^{-.4(5)}} \approx \boxed{111 \text{ students}}$

c. What is the maximum number of students that will be infected?

1600

d. According to this model, when will the number of students infected by 800?

$800 = \frac{1600}{1+99e^{-.4t}}$

$800(1+99e^{-.4t}) = 1600$

$1+99e^{-.4t} = 2$

$99e^{-.4t} = 1$

$e^{-.4t} = \frac{1}{99}$

$-.4t = \ln(\frac{1}{99})$

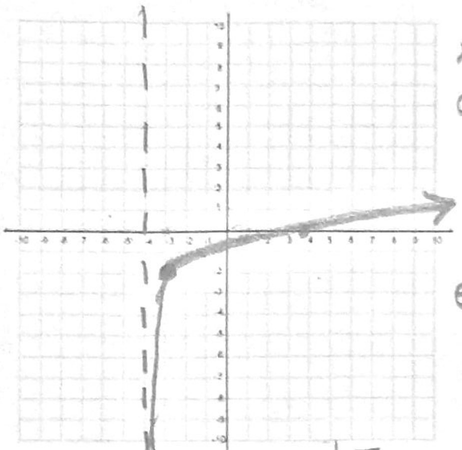
$\boxed{t \approx 11.49 \text{ days}}$

Graph the following equations. Then find the domain, range, asymptote(s), increasing/decreasing, end behavior and x and y intercepts.

15) $f(x) = \ln(x+4) - 2$

Critical Point

$(-3, -2)$



x-int

$0 = \ln(x+4) - 2$

$2 = \ln(x+4)$

$e^2 = x+4$

$e^2 - 4 = x$

Domain: $(-4, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -4$

Increasing: $(-4, \infty)$

Decreasing: —

x-intercept: $(e^2 - 4, 0)$ or $(3.39, 0)$

y-intercept: $(0, \ln(4) - 2)$ or $(0, -.61)$

End behavior:

R.E.B. $\lim_{x \to \infty} f(x) = \infty$ L.E.B. $\lim_{x \to -4} f(x) = -\infty$

Inverse:

$x = \ln(y+4) - 2$

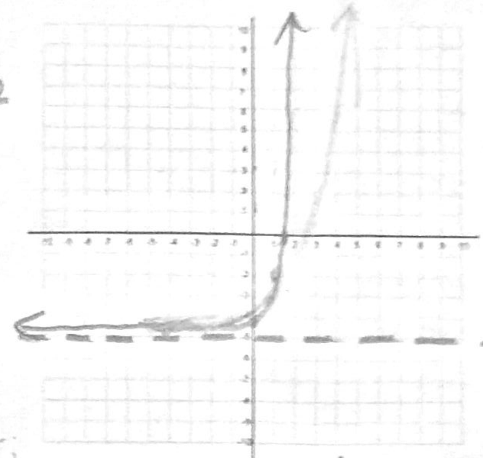
$x + 2 = \ln(y+4)$

$e^{x+2} = y + 4$

$e^{x+2} - 4 = y$

$\boxed{f^{-1}(x) = e^{x+2} - 4}$

16) $f(x) = 3 \cdot 4^{x-1} - 5$



x-int

$0 = 3 \cdot 4^{x-1} - 5$

$5 = 3 \cdot 4^{x-1}$

$\frac{5}{3} = 4^{x-1}$

$\log_4\left(\frac{5}{3}\right) = x - 1$

$\log_4(5/3) + 1 = x$

y-int

$y = 3 \cdot 4^{0-1} - 5$

$y = \frac{3}{4} - 5$

$y = \frac{3}{4} - \frac{20}{4}$

$y = \frac{-17}{4}$

Domain: $(-\infty, \infty)$

Range: $(-5, \infty)$

Asymptote: $y = -5$

Increasing: $(-\infty, \infty)$

Decreasing: —

x-intercept: $(\log_4 \frac{5}{3} + 1, 0)$ or $(1.37, 0)$

y-intercept: $(0, -\frac{17}{4})$

End behavior:

L.E.B. $\lim_{x \to -\infty} f(x) = -5$ R.E.B. $\lim_{x \to \infty} f(x) = \infty$

Inverse: $x = 3(4)^{y-1} - 5$

$x + 5 = 3(4)^{y-1}$

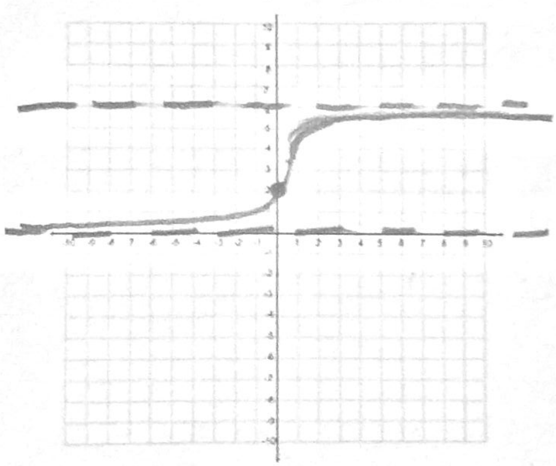
$\frac{1}{3}(x+5) = 4^{y-1}$

$\log_4\left[\frac{1}{3}(x+5)\right] = y - 1$

$\log_4\left[\frac{1}{3}(x+5)\right] + 1 = y$

$\boxed{f^{-1}(x) = \log_4\left[\frac{1}{3}(x+5)\right] + 1}$

17) $f(x) = \frac{8}{1+3\left(\frac{1}{2}\right)^{x}}$



Domain: $(-\infty, \infty)$

Range: $(0, 8)$

Asymptote: $y = 0$ & $y = 8$

Increasing: $(-\infty, \infty)$

Decreasing: —

x-intercept: NONE

y-intercept: $(0, 2)$

End behavior:

$\lim_{x \to \infty} f(x) = 8^-$ $\lim_{x \to -\infty} f(x) = 0^+$