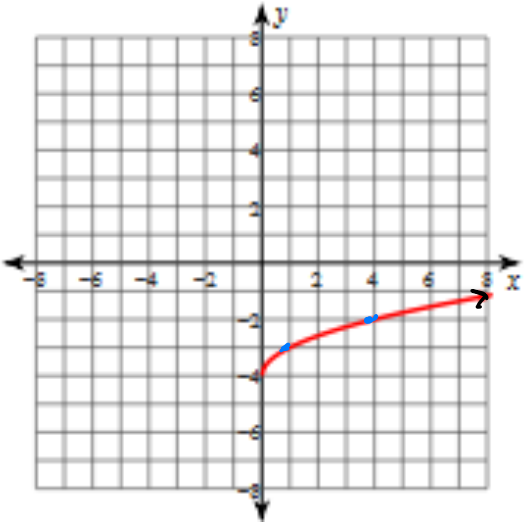
Key

H Math 2
Test 6 Review

1) Equation: $y = \sqrt{x} - 4$



Domain: $[0, \infty)$

Range: $[-4, \infty)$

Interval of increasing: $[0, \infty)$

Interval of decreasing: N/A

x-intercept(s): $(16, 0)$

y-intercept(s): $(0, -4)$

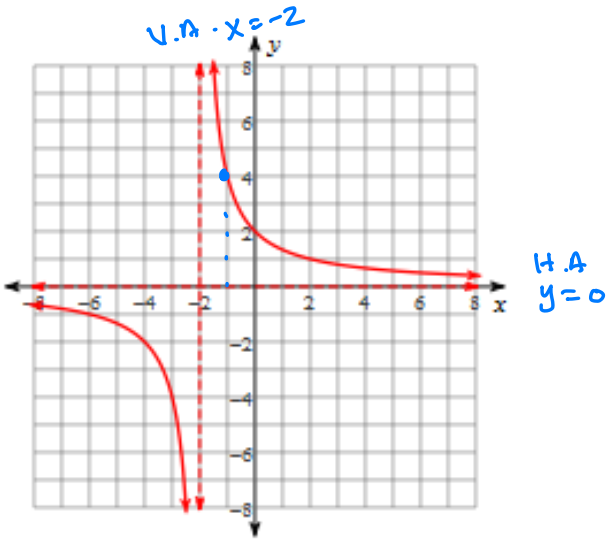
Asymptote(s): none

Transformation(s): shift down 4

End Behavior: R.E.B $x \to \infty, y \to \infty$

x-int (0)
$0 = \sqrt{x} - 4$
$4 = \sqrt{x}$
$16 = x$

2) Equation: $y = \frac{4}{x+2}$



Domain: $(-\infty, -2) \cup (-2, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Interval of increasing: none

Interval of decreasing: $(-\infty, -2) \cup (-2, \infty)$

x-intercept(s): none

y-intercept(s): $(0, 2)$

Asymptote(s): H.A. $y=0$ V.A. $x=-2$

Transformation(s): vertical stretch by 4, shift left 2

End Behavior: R.E.B. $x \to \infty$ $y \to 0$
L.E.B. $x \to -\infty$ $y \to 0$

3) Equation: $y = -2\sqrt{x+3}$

Domain: $[-3, \infty)$

Range: $(-\infty, 0]$

Interval of increasing: none

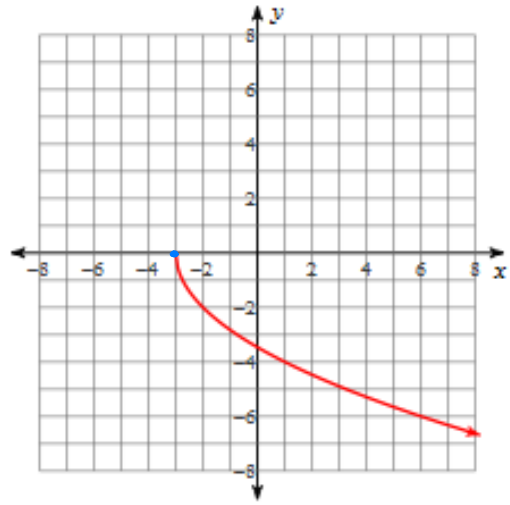
Interval of decreasing: $[-3, \infty)$

x-intercept(s): $(-3, 0)$

y-intercept(s): $(0, -\sqrt{3})$

Asymptote(s): none

y-int
$y = -\sqrt{0+3}$



Transformation(s): • reflect over x-axis, vertical stretch by 2, shift left 3

End Behavior: R.E.B. $x \to \infty$, $y \to -\infty$

4) Equation: $y = -\frac{1}{x+1} - 2$

Domain: $(-\infty, -1) \cup (-1, \infty)$

Range: $(-\infty, -2) \cup (-2, \infty)$

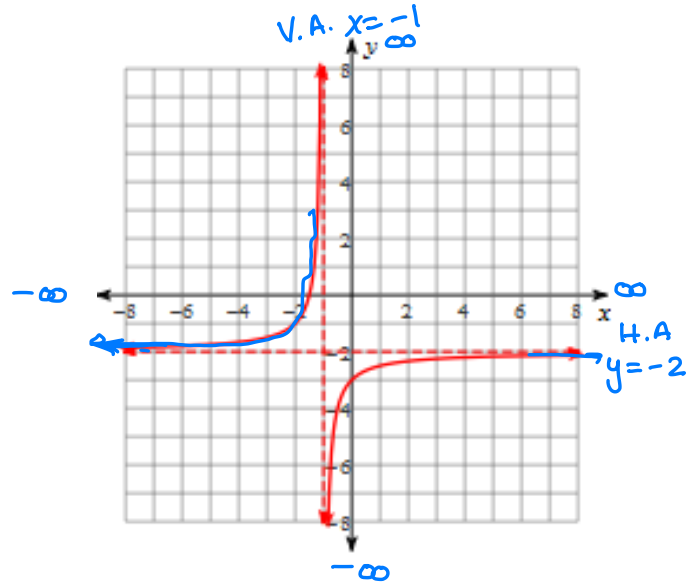
Interval of increasing: $(-\infty, -1) \cup (-1, \infty)$

Interval of decreasing: N/A

x-intercept(s): $(-1.5, 0)$

y-intercept(s): $(0, -3)$

Asymptote(s): H.A $y = -2$ V.A. $x = -1$



Transformation(s): _____

End Behavior: L.E.B $x \to -\infty$ $y \to -2$

R.E.B. $x \to \infty$ $y \to -2$

x-int : (, 0)

$0 = -\frac{1}{x+1} - 2$

$2 = -\frac{1}{x+1}$

$-2 = \frac{1}{x+1}$

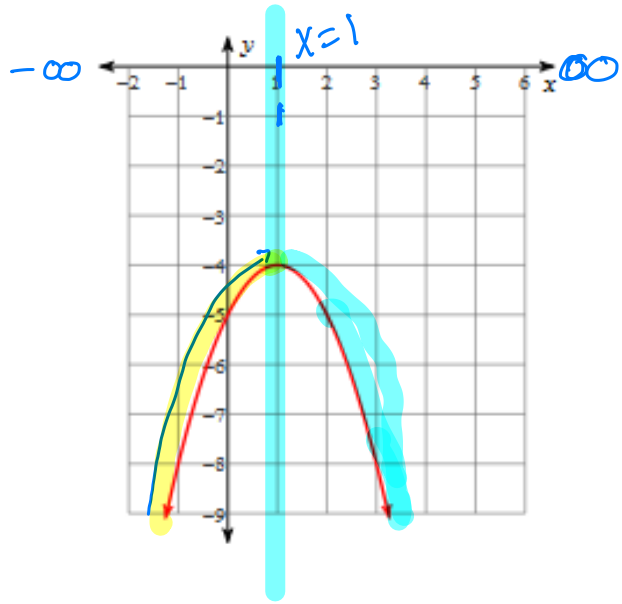
$-2(x+1) = 1$

$-2x - 2 = 1$

$-2x = 3$

$x = -1.5$

5) Equation: $y = -(x-1)^2 - 4$



Domain: $(-\infty, \infty)$

Range: $(-\infty, -4]$

Interval of increasing: $(-\infty, 1)$

Interval of decreasing: $(1, \infty)$

x-intercept(s): none

y-intercept(s): $(0, -5)$

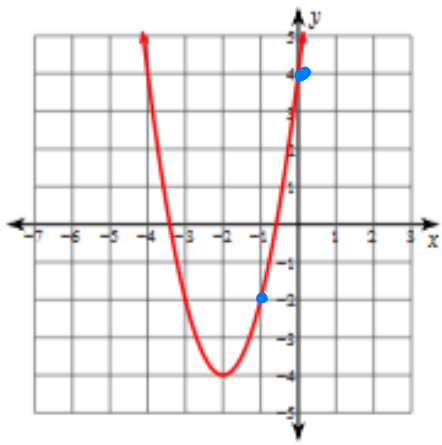
Asymptote(s): none

Transformation(s): Reflect over x-axis, shift right 1, down 4

End Behavior: L.E.B. $x \to -\infty$, $y \to -\infty$

R.E.B. $x \to \infty$, $y \to -\infty$

6) Equation: $y = 2(x+2)^2 - 4$



Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

Interval of increasing: $(-2, \infty)$

Interval of decreasing: $(-\infty, 2)$

x-intercept(s): $(-2+\sqrt{2}, 0)$ & $(-2-\sqrt{2}, 0)$

y-intercept(s): $(0, 4)$

Asymptote(s): none

Transformation(s): vertical stretch by 2, shift left 2, down 4

End Behavior:

L.E.B. $x \to -\infty$, $y \to \infty$

R.E.B. $x \to \infty$, $y \to \infty$

x-int: (0)

$0 = 2(x+2)^2 - 4$

$4 = 2(x+2)^2$

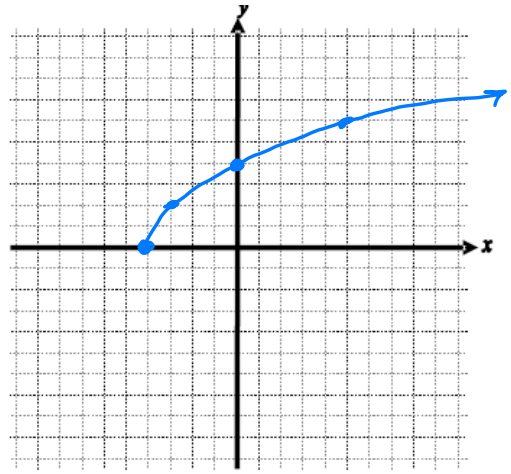
$\sqrt{2} = \sqrt{(x+2)^2}$

$\pm\sqrt{2} = x+2$

$-2 \pm \sqrt{2} = x$

P
E
MD
AS

7) Graph: $y = 2\sqrt{x+4}$



Domain: $[-4, \infty)$

Range: $[0, \infty)$

Interval of increasing: $[-4, \infty)$

Interval of decreasing: N/A

x-intercept(s): $(-4, 0)$

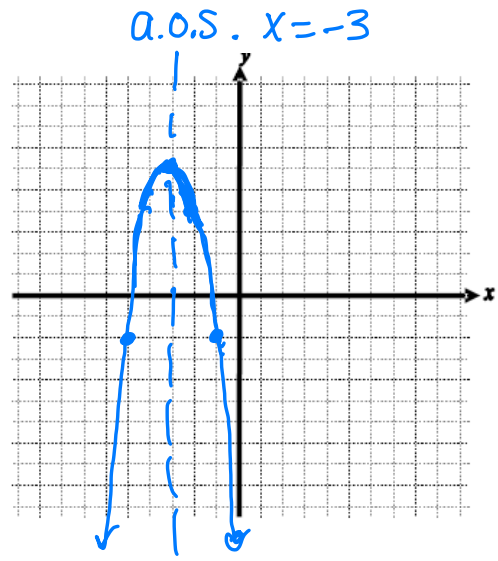
y-intercept(s): $(0, 4)$

Asymptote(s): none

Transformation(s): vertical stretch by 2, shift left 4

End Behavior: R.E.B. $x \to \infty$, $y \to \infty$

8) Graph: $y = -2(x+3)^2 + 6$ Vertex $(-3, 6)$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 6]$

Interval of increasing: $(-\infty, -3)$

Interval of decreasing: $(-3, \infty)$

x-intercept(s): $(-3+\sqrt{3}, 0)$ $(-3-\sqrt{3}, 0)$

y-intercept(s): $(0, -12)$

Asymptote(s): none

Transformation(s): reflect over x-axis, vertical stretch by 2, shift left 3 up 6

End Behavior:

L.E.B $x \to -\infty$, $y \to -\infty$

R.E.B. $x \to \infty$, $y \to -\infty$

y-int $(0, -12)$

$y = -2(0+3)^2 + 6$

$y = -2(9) + 6$

$y = -18 + 6$

$y = -12$

x-int $(\ \ 0)$

$0 = -2(x+3)^2 + 6$

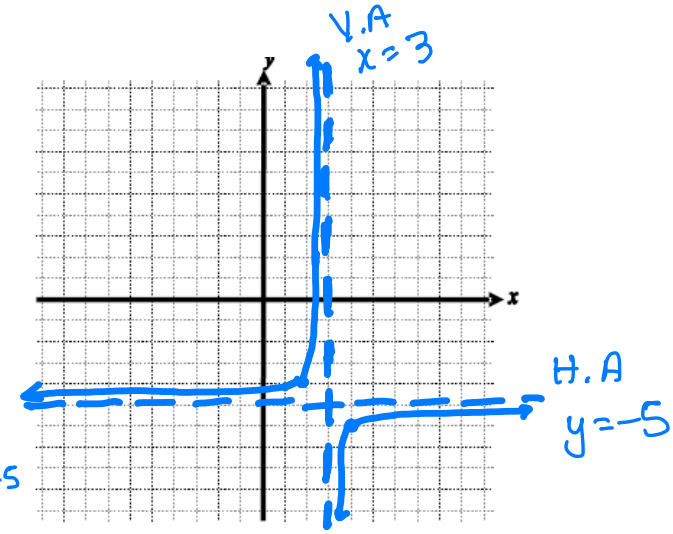
$-6 = -2(x+3)^2$

$3 = (x+3)^2$

$\pm\sqrt{3} = x + 3$

$-3 \pm \sqrt{3} = x$

9) Graph: $f(x) = -\frac{1}{x-3} - 5$



Domain: $(-\infty, 3) \cup (3, \infty)$

Range: $(-\infty, 5) \cup (5, \infty)$

Interval of increasing: $(-\infty, 3) \cup (3, \infty)$

Interval of decreasing: none

x-intercept(s): $(2\frac{4}{5}, 0)$

y-intercept(s): $(0, -4\frac{2}{3})$

$y = -\frac{1}{0-3} - 5$

$y = \frac{1}{3} - 5$

$y = -4\frac{2}{3}$

Asymptote(s): H.A. $y = -5$ V.A $x = 3$

Transformation(s): reflect over x-axis, shift right 3 & down 5

End Behavior: L.E.B. $x \to \infty$, $y \to -5$

R.E.B $x \to -\infty$, $y \to -5$

x-int: (,0)

$0 = -\frac{1}{x-3} - 5$

$5 = -\frac{1}{x-3}$

$5(x-3) = -1$

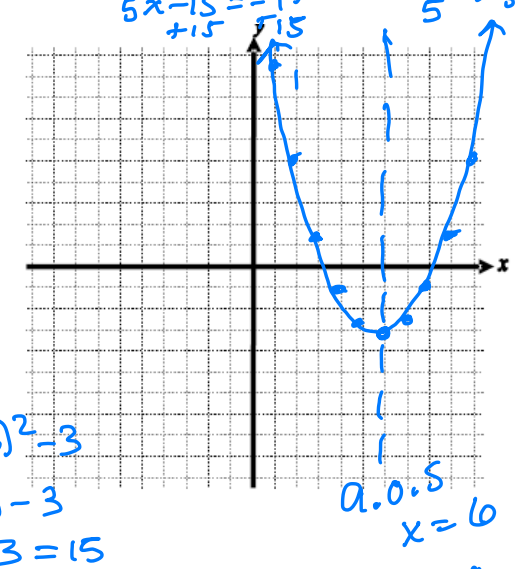
$5x - 15 = -1$

$+15 \quad +15$

$5x = 14$

$x = \frac{14}{5} = 2\frac{4}{5}$

10) Graph: $y = \frac{1}{2}(x-6)^2 - 3$ Vertex $(6, -3)$



Domain: $(-\infty, \infty)$

Range: $[-3, \infty)$

Interval of increasing: $(6, \infty)$

Interval of decreasing: $(-\infty, 6)$

x-intercept(s): $(6+\sqrt{6}, 0)$ & $(6-\sqrt{6}, 0)$

y-intercept(s): $(0, 15)$

$y = \frac{1}{2}(0-6)^2 - 3$

$= \frac{1}{2}(36) - 3$

$= 18 - 3 = 15$

Asymptote(s): none

Transformation(s): vertical shrink by $\frac{1}{2}$, shift right 6 & down 3

End Behavior:

L.E.B $x \to \infty$, $y \to \infty$

R.E.B. $x \to \infty$ $y \to \infty$

x-int: (0)

$0 = \frac{1}{2}(x-6)^2 - 3$

$3 = \frac{1}{2}(x-6)^2$

$\sqrt{6} = \sqrt{(x-6)^2}$

$\pm\sqrt{6} = x - 6$

$6 \pm \sqrt{6} = x$

Solve.

11) $\sqrt{x+10} = 2$

$x + 10 = 4$

$\boxed{x = -6}$

12) $\sqrt{x} = x - 6$

$x = (x-6)^2$

$x = x^2 - 12x + 36$

$0 = x^2 - 13x + 36$

$0 = (x-9)(x-4)$

$x - 9 = 0$ $\quad x - 4 = 0$

$\boxed{x = 9}$ $\quad \cancel{x = 4}$ extraneous

13) $\sqrt{2x+6} - x = -1$

$(\sqrt{2x+6})^2 = (x-1)^2$

$2x + 6 = x^2 - 2x + 1$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x - 5 = 0$ $\quad x + 1 = 0$

$\boxed{x = 5}$ $\quad \cancel{x = -1}$ extraneous

14) $(\sqrt[3]{x-1})^3 = (-3)^3$

$x - 1 = -27$

$\boxed{x = -26}$

15) $\sqrt{x-3} - \sqrt{5x-1} = 0$

$\sqrt{x-3} = \sqrt{5x-1}$

$x - 3 = 5x - 1$

$1 - 3 = 5x - x$

$-2 = 4x$

$\cancel{-\frac{1}{2} = x}$

extraneous

$\boxed{\text{no solution}}$

16) $\frac{2}{x} = x + 1$

$2 = x^2 + x$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x + 2 = 0$ $\quad x - 1 = 0$

$\boxed{x = -2}$ $\quad \boxed{x = 1}$

17) $0.5x + 1 = \frac{4}{x}$

$.5x^2 + x = 4$

$.5x^2 + x - 4 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x + 4 = 0$ $\quad x - 2 = 0$

$\boxed{x = -4}$ $\quad \boxed{x = 2}$

18) $17 + 4\sqrt{x+7} = 1$

$\frac{4\sqrt{x+7}}{4} = \frac{-16}{4}$

$\sqrt{x+7} = -4$

not possible

so, $\boxed{\text{no solution}}$

19) $4x^2 - 5 = 75$

$+5 \quad +5$

$\frac{4x^2}{4} = \frac{80}{4}$

$\sqrt{x^2} = \sqrt{20}$

$4 < 2, 2$; $20 < 5$

$\boxed{x = \pm 2\sqrt{5}}$

20) $x^2 + 4x + 8 = 0$

$x^2 + 4x + 4 = -8 + 4$

$\sqrt{(x+2)^2} = \sqrt{-4}$

$x + 2 = \pm 2i$

$\boxed{x = -2 \pm 2i}$

21) $\sqrt{2x-7} = x - 3$

$2x - 7 = (x-3)^2$

$2x - 7 = x^2 - 6x + 9$

$-2x + 9 \quad -2x + 7$

$0 = x^2 - 8x + 16$

$0 = (x-4)(x-4)$

$x - 4 = 0$

$\boxed{x = 4}$

22) $4x^2 + 4x - 5 = 4$

$4x^2 + 4x - 9 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(4)(-9)}}{8}$

$x = \frac{-4 \pm \sqrt{160}}{8}$

$16 < 4, 4$; $160 < 10, 16$; $2 \cdot 2 \cdot \sqrt{8 \cdot 2}$; $4\sqrt{10}$

$x = \frac{-4 \pm 4\sqrt{10}}{8}$

$\boxed{x = \frac{-1 \pm 1\sqrt{10}}{2}}$

Find the solutions to the following system of equations:

23) $y = x^2 - 3x + 2$
$y = x + 2$

$x + 2 = x^2 - 3x + 2$
$-x - 2 \qquad -x - 2$
$0 = x^2 - 4x$
$0 = x(x-4)$
$x = 0 \qquad x - 4 = 0$
$x = 4$
$y = 0 + 2 \qquad y = 4 + 2$
$y = 2 \qquad y = 6$
$\boxed{(0,2)} \qquad \boxed{(4,6)}$

24) $y = \frac{6}{x+3}$
$y = x - 2$

$(x-2) = \frac{6}{(x+3)}$
$(x-2)(x+3) = 6$
$x^2 + x - 6 = 6$
$-6 \quad -6$
$x^2 + x - 12 = 0$
$(x+4)(x-3) = 0$
$x = -4 \qquad x = 3$
$y = -4 - 2 \qquad y = 3 - 2$
$y = -6 \qquad y = 1$
$\boxed{(-4,-6)} \qquad \boxed{(3,1)}$

25) $y = \frac{1}{2}\sqrt{3x+1}$
$y = x$

$x = \frac{1}{2}\sqrt{3x+1}$
$2x = \sqrt{3x+1}$
$4x^2 = 3x + 1$
$4x^2 - 3x - 1 = 0$
$(4x+1)(x - 1) = 0$
$4x + 1 = 0 \qquad x - 1 = 0$
$\cancel{x = -\frac{1}{4}} \qquad x = 1$
extraneous
$\boxed{(1,1)}$

26) The sum of two numbers is 16 and their product 48. Determine the numbers.

$x + y = 16$
$xy = 48$
$y = \frac{48}{x}$
$x\left(x + \frac{48}{x}\right) = (16)x$
$x^2 + 48 = 16x$
$x^2 - 16x + 48 = 0$
$(x-12)(x-4) = 0$
$x - 12 = 0 \qquad x - 4 = 0$
$x = 12 \qquad x = 4$

$\boxed{4 \text{ and } 12}$

27) The number of gallons of fuel used on a trip varies directly with the number of miles traveled. If a trip of 270 miles required 12 gallons of fuel, how many gallons are required for a trip of 350 miles?

$g = km$
$\frac{12}{270} = \frac{k(270)}{270}$
$\frac{2}{45} = k$

$g = \frac{2}{45}m$
$g = \frac{2}{45}(350) \approx \boxed{15.6g}$

28) The frequency of a vibrating string varies inversely as its length. A string 3 feet long vibrates 175 cycles per second. Find the frequency of a 5 foot string.

$f = \frac{k}{l}$
$175 = \frac{k}{3}$
$525 = k$

$f = \frac{525}{l}$
$f =$

$f = \frac{525}{5}$
$f = \boxed{105 \text{ cycles per second}}$

29) Some students volunteered to clean up a highway near their school. The amount of time it will take varies directly with the length of the section of highway and inversely with the number of students who will help. If 25 students clean up 5 miles of highway, the project will take 2 hours. How long will it take 85 students to clean up 34 miles of highway?

$t = \frac{kl}{n}$
$2 = \frac{k(5)}{25}$
$\frac{\cancel{25}^{5}(2)}{\cancel{5}} = k$
$10 = k$

$t = \frac{10l}{n}$

$t = \frac{10(34)}{85}$
$\boxed{t = 4 \text{ hours}}$

30) The strength of a rectangular beam varies jointly as its width and the square of it depth. If the strength of a beam three inches wide by 10 inches deep is 1200 pounds per square inch, what is the strength of a beam four inches wide and six inches deep?

$B = kwd^2$

$1200 = k(3)(10)^2$

$k = 4$

$B = 4wd^2$

$B = 4(4)(6)^2$

$B = \boxed{576 \text{ psi}}$

31) An egg is dropped from the roof of a building. The distance it falls varies directly with the square of the time it falls. If it takes 0.5 seconds for the egg to fall eight feet, how long will it take the egg to fall 200 feet?

$d = kt^2$

$-8 = k(.5)^2$

$k = -32$

$d = -32t^2$

$\frac{-200}{-32} = \frac{-32t^2}{-32}$

$\sqrt{6.25} = \sqrt{t^2}$

$\boxed{2.5 \text{ seconds}}$

32) Determine the $\boxed{b^2-4ac}$ discriminant and the nature of the roots for each of the following

a) $y = 9x^2 - 30x + 25$

0; 1 real, rational root

b) $y = 6x^2 + 7x - 5$

169; 2 real, rational roots

c) $y = 2x^2 + 8x - 15$

184; 2 real, irrational roots

d) $y = -3x^2 - 7x - 10$

−71; 2 imaginary roots

33) Find the solution to the quadratic inequality. Write your answer in interval form.

$x^2 - 2x - 8 \leq 0$

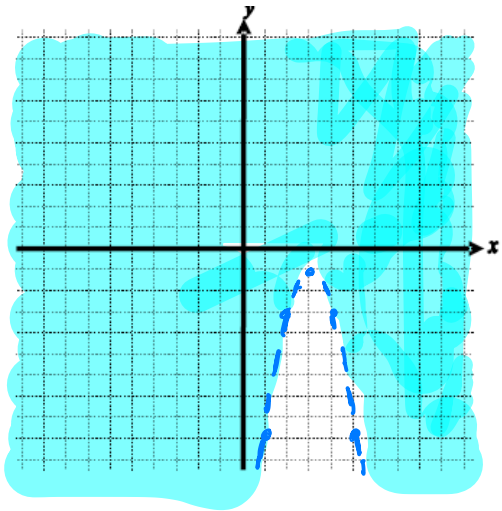
$(x-4)(x+2) \leq 0$

$\boxed{[-2, 4]}$

or

$\boxed{-2 \leq x \leq 4}$

34) Graph: $y > -2(x-3)^2 - 1$



35) Graph: $y > (x+2)(x+5)$

Vertex

$(-3.5, -2.25)$

$(-3.5+2)(-3.5+5)$

$(-1.5)(1.5)$

-2.25