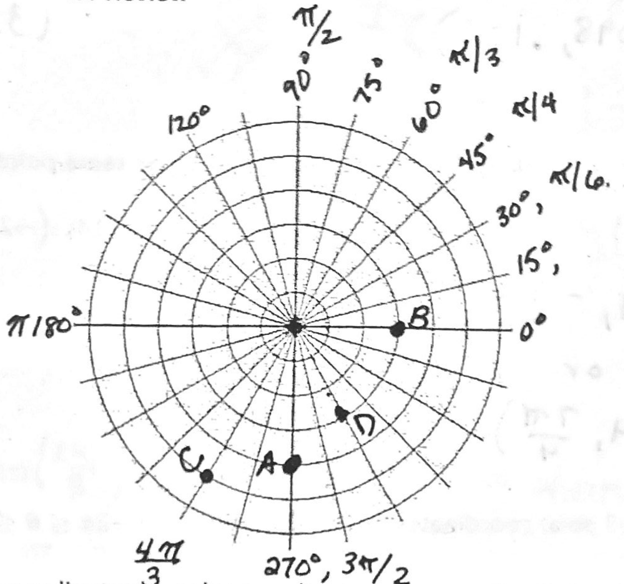
Name: Key

Polar Review

Plot each polar coordinate on the graph.

1. $A(4, 270°)$
2. $B(-3, \pi)$
3. $C\left(5, \frac{4\pi}{3}\right)$
4. $D(-3, 120°)$



Convert each of the following rectangular coordinates to polar coordinates. Answers for θ should be in radians, exact answers if possible. If you need to have decimals, please round to the nearest thousandth.

5. $(-3, 3)$

(r, θ)

$\left(3\sqrt{2}, \frac{3\pi}{4}\right)$ or $\left(-3\sqrt{2}, -\frac{\pi}{4}\right)$

or $\left(-3, \frac{7\pi}{4}\right)$ or $\left(3\sqrt{2}, -\frac{5\pi}{4}\right)$

6. $(-2, -2\sqrt{3})$

$\left(4, \frac{4\pi}{3}\right)$ or $\left(4, -\frac{2\pi}{3}\right)$

or $\left(-4, \frac{\pi}{3}\right)$ $\left(-4, \frac{-5\pi}{3}\right)$

7. $(-0.8, -2.1)$

$\sqrt{(-.8)^2 + (-2.1)^2} = c^2$

$c = \sqrt{5.05} \approx 2.247$

$\tan\theta = \frac{-2.1}{-.8}$ $\quad \theta \approx 69.1°$

$\approx \boxed{(-2.247, 69.1°)}$ or $\boxed{(2.247, 249.1°) \quad (2.247, -110.9°)}$

8. $(-2.3, 0.2)$

$(-2.309, -4.97°)$ or $(2.309, 175.03°)$ $(2.309, -184.97°)$

$(.2)^2 + (-2.3)^2 = r^2$

$r = \sqrt{5.33}$

$r \approx 2.309$

Convert each of the following polar coordinates to rectangular coordinates. Decimals should be rounded to the nearest thousandth.

9. $(5, 300°)$

$(5\cos 300°, 5\sin 300°)$

$\left(\frac{5}{2}, \frac{-5\sqrt{3}}{2}\right)$

10. $\left(4, \frac{3\pi}{2}\right)$

$\left(4\cos\frac{3\pi}{2}, 4\sin\frac{3\pi}{2}\right)$

$\boxed{(0, -4)}$

11. $(-3.1, 182°)$

$(-3.1\cos(182°), -3.1\sin(182°))$

$\approx (3.098, .108)$

12. $(8.1, 5.2)$ ← radians

$(8.1\cos 5.2, 8.1\sin 5.2)$

$(3.795, -7.156)$

Find the other 3 polar coordinates that represent the same point for $-2\pi \le \theta \le 2\pi$.

13. $\left(4, \frac{3\pi}{4}\right)$

$\left(-4, -\frac{\pi}{4}\right)$ or $\left(4, \frac{5\pi}{4}\right)$

or

$\left(-4, \frac{7\pi}{4}\right)$

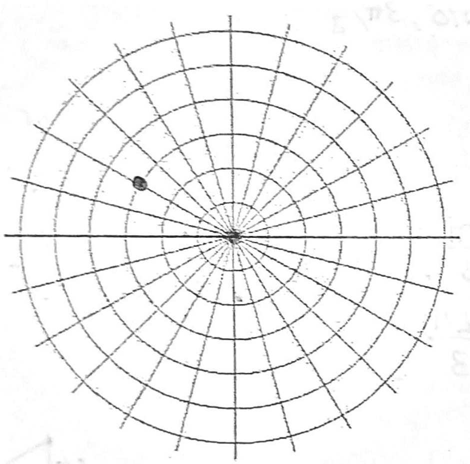
14. $\left(-2, -\frac{2\pi}{3}\right)$

$\left(2, \frac{\pi}{3}\right)$

$\left(2, -\frac{5\pi}{3}\right)$

$\left(-2, \frac{4\pi}{3}\right)$

15. Find all polar coordinates for the point such that $-2\pi \le \theta \le 2\pi$.



$\left(3, \frac{5\pi}{6}\right)$

$\left(-3, -\frac{\pi}{6}\right)$

$\left(3, \frac{7\pi}{6}\right)$

$\left(-3, \frac{11\pi}{6}\right)$

Find the other 3 polar coordinates that represent the same point for $-360° \le \theta \le 360°$.

16. $(3, -20°)$

$(3, 340°)$

$(-3, 160°)$

$(-3, -200°)$

17. $(-4, 103°)$

$(4, -77°)$

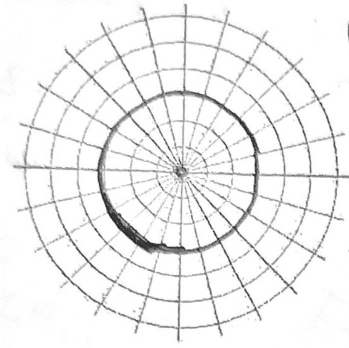
$(4, 283°)$

$(-4, -257°)$

Graph each polar equation. Then convert the equation to rectangular form.

18. $r = 3$

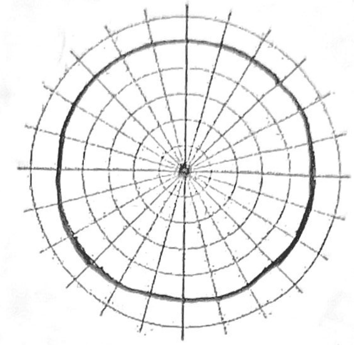
Circle w/ radius of 3



rectangular equation:

19. $r = 5$

Circle w/ radius of 5



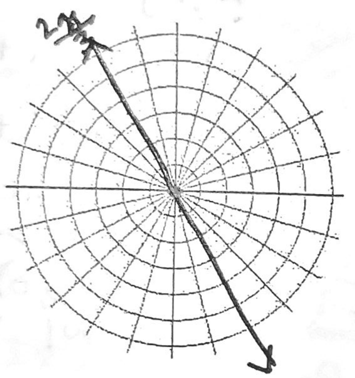
rectangular equation:

18. $\theta = \frac{2\pi}{3}$

$\tan\theta = \tan\left(\frac{2\pi}{3}\right)$

$\frac{y}{x} = -\sqrt{3}$

$\boxed{y = -\sqrt{3}x}$



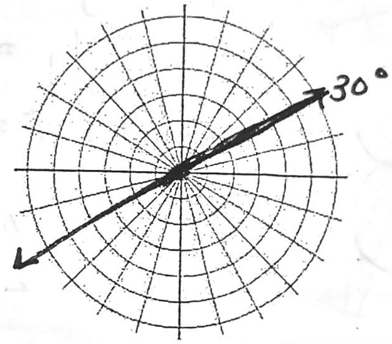
rectangular equation: $\boxed{y = -\sqrt{3}\,x}$

19. $\theta = 30°$

$\tan\theta = \tan 30°$

$\frac{y}{x} = \frac{1}{\sqrt{3}}$

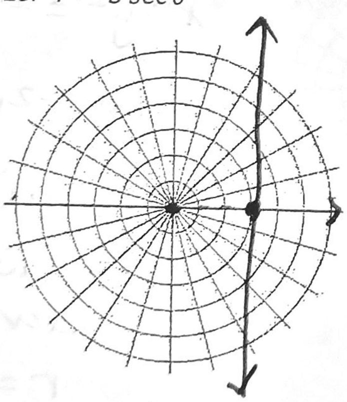
$\boxed{y = \frac{\sqrt{3}}{3}x}$



rectangular equation: $\boxed{y = \frac{\sqrt{3}}{3}x}$

Convert the following polar equations into rectangular equations so that you can graph them.

20. $r = 3\sec\theta$



$r = \frac{3}{\cos\theta}$

$r\cos\theta = 3$

$\boxed{x = 3}$

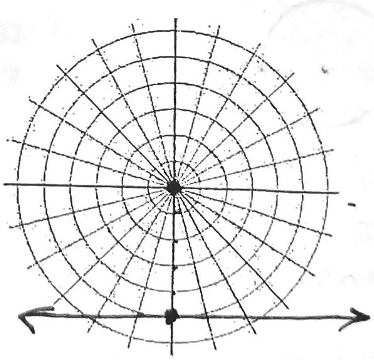
Vertical line

21. $r = -5\csc\theta$

$r = \frac{-5}{\sin\theta}$

$r\sin\theta = -5$

$\boxed{y = -5}$

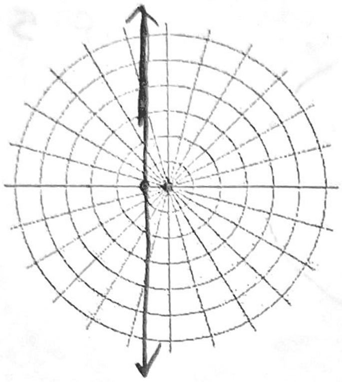


horizontal line

22. $r = -\frac{1}{\cos\theta}$

$r\cos\theta = -1$

$x = -1$

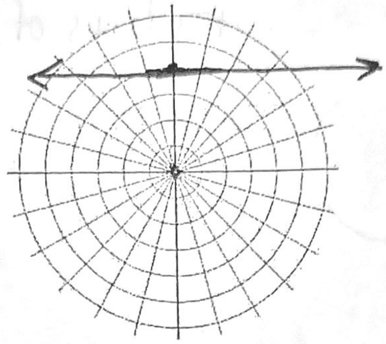


Vertical line

23. $r = \frac{4}{\sin\theta}$

$r\sin\theta = 4$

$\boxed{y = 4}$



horizontal line

24. $r = 4\sin\theta \cdot r$

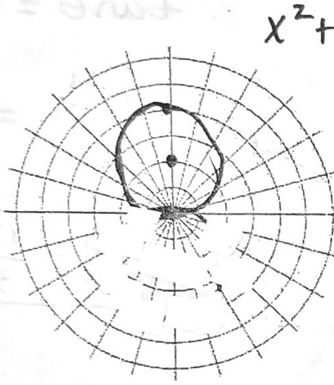
$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$

$x^2 + y^2 - 4y = 0$

$x^2 + (y-2)^2 = 4$

circle center $(0,2)$ $r = 2$



25. $r = -6\cos\theta$

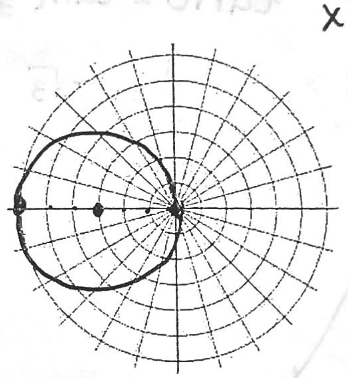
$r^2 = -6r\cos\theta$

$x^2 + y^2 = -6x$

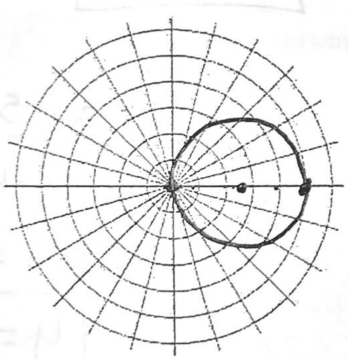
$x^2 + 6x + 9 + y^2 = 0 + 9$

$(x+3)^2 + y^2 = 9$

circle center $(-3,0)$ w/ $r = 3$



25. $r = 5\cos\theta$



$r^2 = 5r\cos\theta$

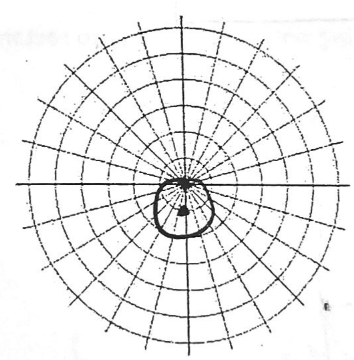
$x^2 + y^2 = 5x$

$x^2 - 5x + \frac{25}{4} + y^2 = \frac{25}{4}$

$(x - \frac{5}{2})^2 + y^2 = \frac{25}{4}$

circle center $(2.5, 0)$ radius 2.5

26. $r = -2\sin\theta$



$r^2 = -2r\sin\theta$

$x^2 + y^2 = -2y$

$x^2 + y^2 + 2y + 1 = 0 + 1$

$x^2 + (y+1)^2 = 1$

Circle w center $(0,-1)$ $r = 1$

Convert each of the following polar equations to rectangular form (general conic form). Classify the conic.

27. $r = (2\sin\theta - 6\cos\theta) \cdot r$

$r^2 = 2r\sin\theta - 6r\cos\theta$

$x^2 + y^2 = 2y - 6x$

$x^2 + y^2 + 6x - 2y = 0$

circle

28. $r = \frac{1}{\cancel{1+\cos\theta}} \cancel{(1+\cos\theta)}$, $(1+\cos\theta)$

$r + r\cos\theta = 1$

$r + x = 1$

$(r)^2 = (1-x)^2$

$\cancel{x^2} + y^2 = 1 - 2x + \cancel{x^2}$

$\frac{y^2 - 1}{-2} = \frac{-2x}{-2}$

parabola

$\boxed{x = -\frac{1}{2}y^2 + \frac{1}{2}}$

29. $r = \frac{3}{5 - 2\sin\theta}$

$5r - 2r\sin\theta = 3$

$5r - 2y = 3$

$(5r)^2 = (3 + 2y)^2$

$25x^2 + 25y^2 = 9 + 12y + 4y^2$

$\boxed{25x^2 + 21y^2 - 12y - 9 = 0}$

Ellipse

30. $r = \frac{2}{1 + 4\cos\theta}$

$r + 4r\cos\theta = 2$

$r + 4x = 2$

$(r)^2 = (2 - 4x)^2$

$x^2 + y^2 = 4 - 16x + 16x^2$

$\boxed{-15x^2 + y^2 + 16x - 4 = 0}$

Hyperbola

Convert each of the following rectangular equations to polar form.

31. $x^2 + y^2 = 4x$

$r^2 = 4r\cos\theta$

$r^2 - 4r\cos\theta$

$r(r - 4\cos\theta) = 0$

$r \neq 0$ $\boxed{r = 4\cos\theta}$

32. $(x-7)^2 + (y+2)^2 = 53$

$x^2 - 14x + \cancel{49} + y^2 + 4y + \cancel{4} = \cancel{53}$

$x^2 + y^2 - 14x + 4y = 0$

$r^2 - 14r\cos\theta + 4r\sin\theta = 0$

$r(r - 14\cos\theta + 4\sin\theta) = 0$

$r \neq 0$ $\boxed{r = 14\cos\theta - 4\sin\theta}$

33. $y = -x$

$\frac{y}{x} = -1$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\boxed{\theta = \frac{7\pi}{4}}$ or $\boxed{\theta = -\frac{\pi}{4}}$ or $\boxed{\theta = \frac{3\pi}{4}}$

34. $y = \frac{\sqrt{3}}{3}x$

$\frac{y}{x} = \frac{\sqrt{3}}{3}$

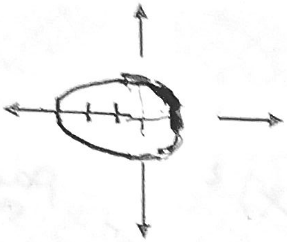
$\tan\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\boxed{\theta = \frac{\pi}{6}}$ or $\boxed{\theta = \frac{7\pi}{6}}$

Identify and sketch a graph for the following.

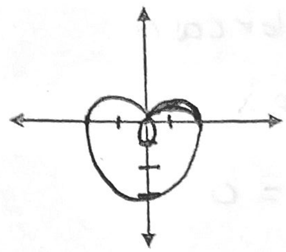
35. $r = 2 - \cos\theta$ Convex

identification: Limaçon



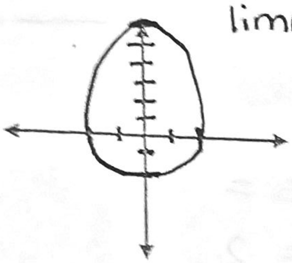
36. $r = 1 - 2\sin\theta$

identification: Limaçon w/ inner loop



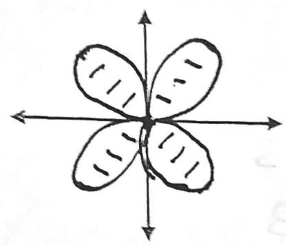
37. $r = 4 + 2\sin\theta$

identification: Convex limaçon



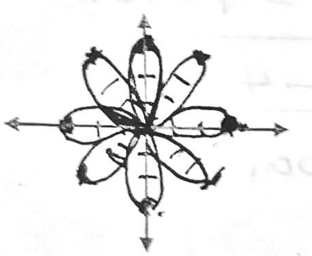
38. $r = 2\sin(2\theta)$

identification: Rose with 4 petals



39. $r = 3\cos(4\theta)$

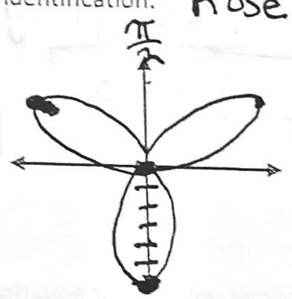
identification: Rose w/ 8 petals



$3\cos(4(0))$
$3 \cdot 1$
3

40. $r = 6\sin(3\theta)$

identification: Rose w/ 3 petals



$\frac{\pi}{2}$

$6\sin\left(\frac{3\pi}{2}\right)$
$6(-1)$
-6

Eliminate the parameter.

24. $x = t - 3,\quad y = 2t + 1,\quad t \geq 0$

$x + 3 = t \quad y = 2(x+3) + 1$

$\boxed{y = 2x + 7}$

line slope 2 y-int (0,7)

25. $x = t + 5,\quad y = \sqrt{t},\quad t \geq 0$

$x - 5 = t$

$\boxed{y = \sqrt{x-5}}$

square root function

26. $x = -2 + t^2,\quad y = 1 + 2t^2,\quad$ *for any t*

$x + 2 = t^2 \quad y = 1 + 2(x+2)$

$y = 1 + 2x + 4$

$\boxed{y = 2x + 5}$

27. $x = e^t,\quad y = t,\quad$ *for any t*

$\ln x = t \quad \boxed{y = \ln x}$

28. $x = 3\cos t,\quad y = 3\sin t,\quad 0 \leq t \leq 2\pi$

$\frac{x}{3} = \cos t \quad \frac{y}{3} = \sin t$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{9} + \frac{y^2}{9} = 1$

$\boxed{x^2 + y^2 = 9}$

29. $x = 2\sin t - 3,\quad y = 2\cos t + 1,\quad 0 \leq t \leq 2\pi$

$\frac{x+3}{2} = \sin t \quad \frac{y-1}{2} = \cos t$

$\cos^2 t + \sin^2 t = 1$

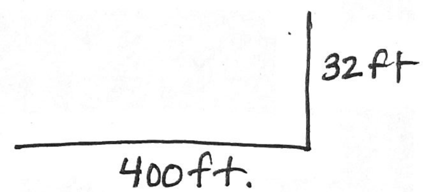
$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{4} = 1$

$\boxed{(x+3)^2 + (y-1)^2 = 4}$

30. A golfer at a driving range stands on a platform 2 feet above the ground and hits the ball with an initial velocity of 120 feet/second at an angle of 39° with the horizontal. There is a 32-foot-high fence 400 feet away. Determine the position of the ball as a pair of parametric equations. Will the ball fall short, hit the fence, or go over the wall?

$x = 120\cos(39°)\,t$

$y = -16t^2 + 120\sin(39°)\,t + 2$



Definite

Degree Mode!!

$400 = 120\cos(39°)\,t$

$t \approx 4.289$

To find vertical height

$y(4.289) \approx 31.55$ ft. No the ball will not clear the wall.

31. Suppose a professional football player kicks a football with an initial velocity of 29 yards per second at an angle of 68° to the horizontal. Suppose a kick returner catches the ball 5 seconds later. Determine the position of the ball as a pair of parametric equations. How far has the ball traveled horizontally and what is its vertical height at the time it is caught?

29 yards = 87 ft.

$$\begin{cases} x = 87\cos(68°)\,t \\ y = -16t^2 + 87\sin(68°)\,t \end{cases}$$

Ball hits the ground after

$x(5) = 87\cos(68°)(5)$

$x(5) = 162.95$

$y(5) = -16(5)^2 + 87\sin(68°)(5)$

$y(5) \approx 3.325$ ft.

The ball traveled 162.95 ft. or 54.3 yards horizontally.
It was caught at a vertical height of 3.32 ft.