Key

Precalculus
9-10 Test Review

$a_n = a_1 + d(n-1)$

1) Write the explicit equation for the arithmetic sequence given $a_{11} = 80$ and $a_{30} = 251$.

$a_n = 80 + 9(n-11)$

$a_n = 80 + 9n - 99$

$\boxed{a_n = 9n - 19}$

$a_{30} = a_{11} + d(30-11)$

$251 = 80 + d(19)$

$d = 9$

2) Write the explicit equation for the geometic sequence given $a_2 = -15$ and $a_5 = -1875$.

$\boxed{a_n = -15(5)^{n-2}}$

or

$\boxed{a_n = -3(5)^{n-1}}$

$a_n = a_1(r)^{n-1}$

$a_5 = a_2(r)^{5-2}$

$-1875 = -15(r)^3$

$125 = r^3$ $r = 5$

3) Find the next 2 terms in the sequence. Then write the recursive and explicit formulas.

a) 4, 9, 14, . . . 19, 24 arithmetic (+5 +5)

recursive

$\begin{cases} a_1 = 4 \\ a_n = a_{n-1} + 5 \end{cases}$

explicit

$a_n = 4 + 5(n-1)$

or

$a_n = 5n - 1$

b) 1000, 800, 640, . . . geometric (×.8 ×.8)

$1000r = 800$

$r = .8$

recursive

$\begin{cases} a_1 = 1000 \\ a_n = .8a_{n-1} \end{cases}$

explicit

$a_n = 1000(.8)^{n-1}$

4) Write the following series in summation notation. Then Evaluate the series.

a) $-5 - 1 + 3 + 7 + 11 + \ldots n = 11$ (+4 +4 +4)

$\sum_{n=1}^{11} (4n-5)$

b) $2, 6, 18, \ldots n = 11$

$2 + 6 + 18$

$\sum_{n=1}^{11} 2(3)^{n-1}$

c) $64, 16, 4, + \ldots$

$64 + 16 + 4 + \ldots$

$\sum_{n=1}^{\infty} 64\left(\frac{1}{4}\right)^{n-1}$

d) $2, 5, 10, 17, 26 + \ldots n = 7$

$2 + 5 + 10 + 17 + 26 + \ldots n = 7$

1st $+3$ $+5$ $+7$ $+9$ ← Quadratic

2nd $+2$ $+2$ $+2$ ← 2nd diff. constant

$\sum_{n=1}^{7} (n^2+1)$

5) Find the number of terms in the series given the following.

a) $a_1 = 18,\ d = 3,\ S_n = 363$

$S_n = \frac{n}{2}(a_1 + a_n)$ $a_n = a_1 + d(n-1)$

$363 = \frac{n}{2}(18 + a_n)$ $a_n = 18 + 3(n-1)$

$a_n = 3n + 15$

$363 = \frac{n}{2}(18 + 3n + 15)$

$726 = 3n^2 + 33n$

$0 = 3n^2 + 33n - 726$

$n = \frac{-33 \pm \sqrt{33^2 - 4(3)(-726)}}{6}$

$\boxed{n = 11}, \cancel{-22}$

b) $a_1 = 2,\ r = 4,\ S_n = 10922$

$S_n = \frac{a_1(1 - r^n)}{1 - r}$

$10922 = \frac{2(1 - 4^n)}{1 - 4}$

$-32766 = 2(1 - 4^n)$

$-16383 = (1 - 4^n)$

$+16384 = +4^n$

$n = \log_4 16384$ or $\frac{\ln(16384)}{\ln 4} = \boxed{7}$

6) Find the sum of the series.

a) $a_n = (-1)^{n-1} \cdot 3a_{n-1}$

for 4 terms given $a_1 = 3$

$a_1 = 3$

$a_2 = (-1)^{2-1} \cdot 3(a_1) = -9$

$a_3 = (-1)^{3-1} \cdot 3(a_2) = -27$

$a_4 = (-1)^{4-1} \cdot 3(a_3) = 81$

$3 + -9 + -27 + 81$

$\boxed{48}$

b) $\sum_{n=3}^{5} (2^{n-1} + 4n)$

$a_3 = 2^{3-1} + 4(3) = 4 + 12 = 16$

$a_4 = 2^{4-1} + 4(4) = 8 + 16 = 24$

$a_5 = 2^{5-1} + 4(5) = 16 + 20 = 36$

$16 + 24 + 36 = \boxed{76}$

c) $\sum_{n=1}^{8} 243\left(\frac{1}{3}\right)^{n-1}$ ← Exponential so, Geometric

$S_n = \frac{a_1(1 - r^n)}{1 - r}$

$S_8 = \frac{243(1 - (1/3)^8)}{1 - 1/3} = \boxed{\frac{3280}{9} \text{ or } 364.\overline{4}}$

d) $\sum_{n=3}^{15} (4n - 3)$ ← Linear so, Arithmetic

$a_3 = 4(3) - 3 = 9$

$a_{15} = 4(15) - 3 = 57$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{13}{2}(a_3 + a_{15})$

$S_n = \frac{13}{2}(9 + 57) = 429$

7) Determine is the series converges. If so, find the sum.

a) $\sum_{k=1}^{\infty} 4\left(\frac{3}{4}\right)^{k-1}$

Converges $|r| < 1$

$S = \frac{a_1}{1 - r}$

$S = \frac{4}{1 - 3/4}$

$= \frac{4}{\frac{1}{4}} = \boxed{16}$

b) $\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$

$\times\frac{1}{2}$ Converges

$S = \frac{\frac{1}{8}}{1 - 1/2}$

$= \frac{\frac{1}{8}}{\frac{1}{2}} \cdot 2$

$= \boxed{\frac{1}{4}}$

c) $\sum_{k=1}^{\infty} 3\left(\frac{5}{2}\right)^{k}$

0th term → 0

$a_1 = 3\left(\frac{5}{2}\right)^1$

$a_1 = \frac{15}{2}$

$\cancel{S = \frac{a_1}{1 - r}}$

$\frac{5}{2} > 1$

$\boxed{\text{diverges, no sum}}$ → ∞

8) A

8) A ball is dropped from 25 feet off the ground. The ball rebounds 70% of its previous height after each bounce.

25ft ↓ ↑17.5↓ $n = \#$ of bounces

a) Write an explicit rule that models the context.

$$a_n = 25(.70)^n$$

b) How high does the ball rebound after the sixth bounce?

$$a_6 = 25(.70)^6 \approx \boxed{2.94 \text{ ft.}}$$

c) How many bounces will occur before the ball rebounds less than a foot?

$$1 = 25(.7)^n \qquad \frac{1}{25} = .7^n \qquad \log_{.7}\left(\frac{1}{25}\right) = n \qquad n \approx 9.0247$$

$\boxed{\text{after the 10th bounce}}$

d) What is the total distance the ball travels before it rolls away?

$$S_D + S_U$$

$$\frac{25}{1-.7} + \frac{17.5}{1-.7}$$

$$\frac{250}{3} + \frac{175}{3} = \frac{425}{3} = \boxed{141\tfrac{2}{3} \text{ ft.}}$$

9) Expand the following;

a) $(3x^2 - 1)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$= 1(3x^2)^5(-1)^0 + 5(3x^2)^4(-1)^1 + 10(3x^2)^3(-1)^2 + 10(3x^2)^2(-1)^3 + 5(3x^2)^1(-1)^4 + 1(3x^2)^0(-1)^5$$

$$= \boxed{243x^{10} - 405x^8 + 270x^6 - 90x^4 + 15x^2 - 1}$$

b) $(4a^4 + b)^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$= 1(4a^4)^4(b)^0 + 4(4a^4)^3(b)^1 + 6(4a^4)^2(b)^2 + 4(4a^4)^1(b)^3 + 1(4a^4)^0(b)^4$$

$$= \boxed{256a^{16} + 256a^{12}b + 96a^8b^2 + 16a^4b^3 + b^4}$$

10) Find the third term in $(2b - 3)^4$.

$${}_4C_2\,(2b)^2(-3)^2$$

$$6\,(4b^2)(9) = \boxed{216b^2}$$

11) Find the coefficient of y^2 in $(4 - y)^3$

$$1(4)^3(-y)^0 + 3(4)^2(-y)^1 + 3(4)^1(-y)^2 + 1(4)^0(-y)^3$$

$\boxed{12}y^2$

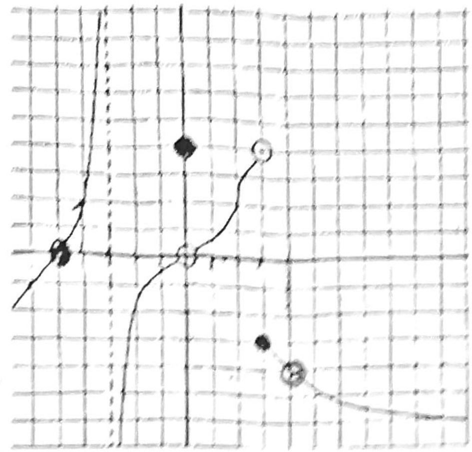
12) Find the 5th term in $(1 + 4y^3)^{10}$ =

$${}_{10}C_4\,(1)^6(4y^3)^4$$

$$210 \cdot 1 \cdot 256y^{12}$$

$$\boxed{53760y^{12}}$$

Evaluate.



13) $f(0) =$ 4

14) $\lim_{x\to 0} f(x) =$ 0

15) $\lim_{x\to -3^+} f(x) = -\infty$

16) $\lim_{x\to -3^-} f(x) = \infty$

17) $\lim_{x\to -3} f(x) =$ DNE

18) $\lim_{x\to 3^+} f(x) = -3$

19) $\lim_{x\to 3^-} f(x) =$ 4

20) $\lim_{x\to 3} f(x) =$ DNE

21) $f(-3) =$ und.

22) $f(3) = -3$

23) $f(4) =$ undefined

24) $\lim_{x\to 4} f(x) = -4$

25) $\lim_{x\to -4^-} f(x) =$ 2

26) $\lim_{x\to -4^+} f(x) =$ 2

27) $\lim_{x\to -4} f(x) =$ 2

28) $f(-4) =$ 2

Evaluate each limit.

29) $\lim_{x\to -2} -\frac{x^2-4}{x+2} = \frac{-(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}$

$= -(x-2)$

30) $\lim_{x\to 4} \frac{(\sqrt{x}-2)}{x-4} \cdot \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \frac{\cancel{(x-4)}\ 1}{\cancel{(x-4)}(\sqrt{x}+2)}$

$= \frac{1}{(\sqrt{4}+2)} = \boxed{\frac{1}{4}}$

31) $\lim_{x\to \infty} \frac{3-4x^2}{x^2+3x+2} = \boxed{-4}$

32) $\lim_{x\to 2} \frac{x-2}{x^3-8} = \frac{\cancel{(x-2)}\ 1}{\cancel{(x-2)}(x^2+2x+4)}$

$= \frac{1}{2^2+2(2)+4} = \boxed{\frac{1}{12}}$

33) $\lim_{x\to -1} \left(-\frac{x^2}{2} - 3x + \frac{1}{2}\right) = \boxed{3}$

Direct Substitution

$\left(-\frac{(-1)^2}{2} - 3(-1) + \frac{1}{2}\right)$

$\left(-\cancel{\tfrac{1}{2}} + 3 + \cancel{\tfrac{1}{2}}\right)$

34) $\lim_{x\to -3} \frac{x}{\frac{1}{3+x} - \frac{1}{3}} = \boxed{0}$

$x \div \left(\frac{1}{x+3} - \frac{1}{3}\right)$

$x \div \left(\frac{3-(x+3)}{3(x+3)}\right)$

$x \div \frac{-x}{3(x+3)}$

$x \cdot \frac{3(x+3)}{-x} = -3(x+3)$

$-3(-3+3)$

$-3(0) = \boxed{0}$

35) If $f(x) = \begin{cases} 2x-9, & x \le 3 \\ -1, & x > 3 \end{cases}$, evaluate each of the following:

a) $\lim_{x\to 3^+} f(x) = -1$

b) $\lim_{x\to 3^-} f(x) = 2(3)-9$
$= \boxed{-3}$

c) $\lim_{x\to 3} f(x) = \boxed{\text{DNE}}$

d) $f(3) = 2(3)-9$
$= \boxed{-3}$

e) $f(5) = \boxed{-1}$

f) $f(-2) = 2(-2)-9$
$= -4-9$
$= \boxed{-13}$