

PreCalculus

Unit 5: Analytic Trig Review
Solve each equation.Name Key

Date _____

1) $4 \cos^2 x - 3 = 0$

$$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$\cos x = -\frac{\sqrt{3}}{2} \quad \cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{5\pi}{6} + k\pi \quad x = \frac{7\pi}{6} + k\pi, \quad k \in \mathbb{Z}$$

3) $\sin(3\theta) = -1, \quad 0 \leq \theta < 2\pi$

$$3\theta = \frac{3\pi}{2} + 2k\pi \quad \theta = \frac{3\pi}{6} + \frac{4k\pi}{3}$$

$$\theta = \frac{\pi}{2} + \frac{2k\pi}{3} \quad \theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

5) $\sin\left(\frac{\theta}{2}\right) - 1 = 0$

$$\sin\left(\frac{\theta}{2}\right) = 1 \quad \theta = \pi + 2k\pi$$

7) $4 \cos^2 x + 4 \cos x - 3 = 0, \quad 0 \leq x < 2\pi$

$$(2\cos x - 1)(2\cos x + 3) = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = -\frac{3}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

 \emptyset

9) $\sin \theta = -0.2, \quad 0 \leq \theta < 2\pi$

$$\theta = -0.201 + 2\pi = 6.082$$

$$\theta = 0.201 + \pi = 3.343$$

2) $2 \sin^2 x - \sin x - 3 = 0, \quad 0 \leq x < 2\pi$

$$(2\sin x - 3)(\sin x + 1) = 0$$

$$\sin x = \frac{3}{2} \quad \sin x = -1$$

 \emptyset

$$x = \frac{3\pi}{2}$$

4) $\sqrt{3} + \tan(2\theta) = 0, \quad 0 \leq x < 2\pi$

$$\tan(2\theta) = -\sqrt{3}$$

$$2\theta = \frac{2\pi}{3} + k\pi$$

$$\theta = \frac{\pi}{3} + \frac{k\pi}{2}, \quad k \in \mathbb{Z}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{6}, \frac{8\pi}{6}, \frac{11\pi}{6}$$

6) $\sec\left(x - \frac{\pi}{5}\right) = 2, \quad 0 \leq x < 2\pi$

$$x - \frac{\pi}{5} = \frac{\pi}{3} + 2k\pi$$

$$x = \frac{5\pi}{15} + \frac{3\pi}{15} + \frac{30k\pi}{15}$$

$$x = \frac{8\pi}{15} + \frac{30k\pi}{15}$$

$$x = \frac{28\pi}{15}$$

8) $2\tan^2 x - \tan x - 6 = 0, \quad 0 \leq x < 2\pi$

$$(2\tan x + 3)(\tan x - 2) = 0$$

$$\tan x = -\frac{3}{2} \quad \tan x = 2$$

$$x = -0.983 + k\pi \quad x = 1.107 + k\pi$$

$$x = 2.159$$

$$5.300$$

$$1.107$$

$$4.249$$

10) $\tan x = 5, \quad 0 \leq x < 2\pi$

$$x = \tan^{-1}(5)$$

$$x \approx 1.373$$

$$x \approx 1.373 + \pi \approx 4.515$$

Determine the exact value of each trigonometric function using a sum/difference and then again using a half/double angle to verify.

11) $\cos \frac{7\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) \quad (12) \tan\left(-\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$$\frac{\pi}{6} = \frac{2\pi}{12}$$

$$\frac{\pi}{4} = \frac{3\pi}{12}$$

$$\frac{\pi}{4} = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{3}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}$$

$$= \frac{1 - \sqrt{3}}{1 + 1 \cdot \sqrt{3}}$$

$$= \frac{(1 - \sqrt{3}) \cdot (1 - \sqrt{3})}{(1 + \sqrt{3})(1 - \sqrt{3})}$$

$$= \frac{1 - 2\sqrt{3} + 3}{-2}$$

$$= \frac{4 - 2\sqrt{3}}{-2} = -2 + \sqrt{3}$$

$$\text{or } \cos\left(\frac{7\pi}{12}\right) = -\frac{\sqrt{2} - \sqrt{6}}{2}$$

13) $\sin \frac{5\pi}{12}$

$$\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\text{or } \tan\left(\frac{5\pi}{12}\right)$$

$$+ \frac{\sqrt{2} + \sqrt{3}}{2}$$

Prove each trigonometric identity.

$$14) \tan \theta \cot \theta - \sin^2 \theta = \cos^2 \theta$$

$$1 - \sin^2 \theta =$$

$$\cos^2 \theta =$$

Q.E.D,

$$15) 4\cos^2 \theta + 3\sin^2 \theta = 3 + \cos^2 \theta$$

$$4\cos^2 \theta + 3(1 - \cos^2 \theta) =$$

$$4\cos^2 \theta + 3 - 3\cos^2 \theta =$$

$$3 + \cos^2 \theta =$$

Q.E.D



$$16) \frac{\csc \theta}{1 + \csc \theta} = \frac{1 - \sin \theta}{\cos^2 \theta}$$

$$\frac{\csc \theta (1 - \csc \theta)}{(1 + \csc \theta)(1 - \csc \theta)} =$$

$$\frac{\csc \theta (1 - \csc \theta)}{1 - \csc^2 \theta} =$$

$$\frac{\frac{1}{\sin \theta} (1 - \frac{1}{\sin \theta})}{1 - \frac{1}{\sin^2 \theta}} =$$

$$18) \frac{\sin \theta + \cos \theta}{\sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta} = \sec \theta \csc \theta$$

$$(\sin \theta + \cos^2 \theta) - (\cos \theta - \sin^2 \theta)$$

$$\sin \theta \cos \theta$$

$$\frac{\cancel{\sin \theta \cos \theta} + \cancel{(\cos^2 \theta)} - \cancel{\sin \theta \cos \theta} \cancel{\sin^2 \theta}}{\cancel{\sin \theta \cos \theta}} =$$

$$\frac{1 + \cancel{\sin^2 \theta}}{\cos \theta \sin \theta} = \sec \theta \csc \theta \checkmark$$

$$20) \cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta$$

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta + \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$2 \cos \alpha \cos \beta \checkmark$$

$$= \cos(2\theta)$$

$$1 + \sin(2\theta)$$

$$15) 4\cos^2 \theta + 3\sin^2 \theta = 3 + \cos^2 \theta$$

$$4\cos^2 \theta + 3(1 - \cos^2 \theta) =$$

$$4\cos^2 \theta + 3 - 3\cos^2 \theta =$$

$$3 + \cos^2 \theta =$$

Q.E.D



$$17) \sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$$

$$= \tan^2 \theta (\tan^2 \theta + 1)$$

$$= \tan^2 \theta (\sec^2 \theta)$$

$$= (\sec^2 \theta - 1)(\sec^2 \theta)$$

$$= \sec^4 \theta - \sec^2 \theta \checkmark$$

$$\frac{1}{\sin \theta} \cdot \frac{\sin^2 \theta}{\sin^2 \theta - 1} = \frac{1/(1 - \sin \theta)}{1/(1 - \sin^2 \theta)} = \frac{1 - \sin \theta}{\cos^2 \theta} \checkmark$$

$$19) \sin(\alpha - \beta) \sin(\alpha + \beta) = \sin^2 \alpha - \sin^2 \beta$$

$$(\sin \alpha \cos \beta - \cos \alpha \sin \beta)(\sin \alpha \cos \beta + \cos \alpha \sin \beta)$$

$$= \sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta$$

$$= \sin^2 \alpha (1 - \sin^2 \beta) - (1 - \sin^2 \alpha) \sin^2 \beta$$

$$= \sin^2 \alpha - \sin^2 \alpha \sin^2 \beta - \sin^2 \beta + \sin^2 \alpha \sin^2 \beta$$

$$= \sin^2 \alpha - \sin^2 \beta \checkmark$$

$$21) \frac{\cos(2\theta)}{1 + \sin(2\theta)} = \frac{\cot \theta - 1}{\cot \theta + 1}$$

$$= \left(\frac{\cos \theta}{\sin \theta} - 1 \right) : \left(\frac{\cos \theta}{\sin \theta} + 1 \right)$$

$$= \frac{\cos \theta - \sin \theta}{\sin \theta} : \frac{\cos \theta + \sin \theta}{\sin \theta}$$

$$= \frac{\cos \theta - \sin \theta}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta + \sin \theta}$$

$$= \frac{(\cos \theta - \sin \theta) \cdot (\cos \theta + \sin \theta)}{(\cos \theta + \sin \theta) \cdot (\cos \theta + \sin \theta)} \checkmark$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + 2 \cos \theta \sin \theta + \sin^2 \theta} = \frac{\cos 2\theta}{1 + \sin(2\theta)}$$

$$22) \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4 \tan\theta \sec\theta$$

$$\begin{aligned} &= \frac{(1+\sin\theta)(1+\sin\theta) - (1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} \\ &= \frac{(1+2\sin\theta+\sin^2\theta) - (1-2\sin\theta+\sin^2\theta)}{1-\sin^2\theta} \\ &= \frac{1+2\sin\theta+\sin^2\theta - 1+2\sin\theta-\sin^2\theta}{\cos^2\theta} \\ &= \frac{4\sin\theta}{\cos\theta \cdot \cos\theta} = 4\tan\theta \sec\theta \quad \checkmark \end{aligned}$$

$$23) \frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x} \cdot \frac{(1-\sin x)}{(1-\sin x)}$$

$$\begin{aligned} &= \frac{\cos x(1-\sin x)}{1-\sin^2 x} \\ &= \frac{\cos x(1-\sin x)}{\cos^2 x} \\ &= \frac{1-\sin x}{\cos x} \quad \checkmark \end{aligned}$$

$$24) \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta}$$

$$\begin{aligned} &= \left(\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} \right) \div \left(\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B} \right) \\ &= \left(\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \right) \div \left(\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} \right) \\ &= \frac{\sin(A+B)}{\cos A \cos B} \cdot \frac{\cos A \cos B}{\sin(A-B)} \\ &= \frac{\sin(A+B)}{\sin(A-B)} \quad \checkmark \end{aligned}$$

$$25) \frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$$

$$\begin{aligned} &\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \right) \div \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \right) = \\ &\left(\frac{\cos^2\theta - \sin^2\theta}{\sin\theta \cos\theta} \right) \div \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cos\theta} = \\ &\frac{\cos^2\theta - \sin^2\theta}{\sin\theta \cos\theta} \cdot \frac{\sin\theta \cos\theta}{\cos^2\theta + \sin^2\theta} = \\ &\cos^2\theta - \sin^2\theta = \cos 2\theta \quad \checkmark \end{aligned}$$

$$26) \tan\frac{\theta}{2} = \csc\theta - \cot\theta$$

$$\begin{aligned} &= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \\ &= \frac{1-\cos\theta}{\sin\theta} \\ &= \tan\left(\frac{\theta}{2}\right) \quad \checkmark \end{aligned}$$

$$27) \sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$$

$$\begin{aligned} &= \sin\left(\frac{3\pi}{2}\right)\cos\theta + \cos\left(\frac{3\pi}{2}\right)\sin\theta \\ &= -1\cos\theta + 0\sin\theta \\ &= -\cos\theta \quad \checkmark \end{aligned}$$

$$28) \frac{\sin\left(x-\frac{\pi}{2}\right)}{\cos\left(x-\frac{\pi}{2}\right)} = -\cot x$$

$$\begin{aligned} &\frac{\sin\left(-\left(\frac{\pi}{2}-x\right)\right)}{\cos\left(-\left(\frac{\pi}{2}-x\right)\right)} = \frac{-\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)} \\ &= -\frac{\cos x}{\sin x} \\ &= -\cot x \quad \checkmark \end{aligned}$$

$$29) \sin^2 x + 2\cot^2\left(\frac{\pi}{2}-x\right) + \frac{1}{\csc x \sin x} = 2\sec^2 x$$

$$\begin{aligned} &\sin^2 x + 2\tan^2 x + \cos^2 x + 1 = \\ &(\sin^2 x) + 2\tan^2 x + (\cos^2 x) + 1 \\ &1 + 2\tan^2 x \\ &2(1 + \tan^2 x) = 2\sec^2 x \quad \checkmark \end{aligned}$$

Solve each triangle.

30) $B = 82^\circ, b = 17, c = 15$ Δ Law of Sines

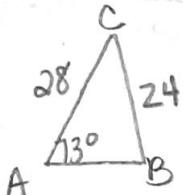


$$\frac{\sin 82^\circ}{17} = \frac{\sin C}{15}$$

$$\begin{aligned}\angle C &\approx 60.9^\circ \\ \angle A &\approx 37.1^\circ \\ a &= 10.36\end{aligned}$$

$$\frac{\sin 82^\circ}{17} = \frac{\sin A}{a}$$

32) $A = 73^\circ, a = 24, b = 28$



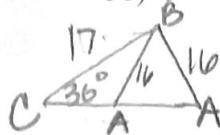
$$\frac{\sin 73^\circ}{24} = \frac{\sin B}{28}$$

not possible

34) $a = 12, b = 15, c = 28$

$12 + 15 < 28$
not possible

31) $c = 36^\circ, a = 17, c = 16$



$$\frac{\sin 36^\circ}{16} = \frac{\sin A}{17}$$

when $\angle A$ is acute

$$\begin{aligned}\angle A &\approx 38.6^\circ \\ \angle B &\approx 105.4^\circ \\ b &\approx 26.25\end{aligned}$$

when $\angle A$ is obtuse

$$\begin{aligned}\angle A &\approx 141.4^\circ \\ \angle B &\approx 2.6^\circ \\ b &\approx 1.26\end{aligned}$$

$$\frac{\sin 36^\circ}{16} = \frac{\sin B}{b}$$

33) $a = 14, b = 17, c = 25$

$$25^2 = 14^2 + 17^2 - 2(14)(17)\cos C$$

$$\frac{(25^2 - 14^2 - 17^2)}{-2(14)(17)} = \cos C$$

$$\angle C \approx 107.1^\circ$$

$$17^2 = 14^2 + 25^2 - 2(14)(25)\cos B$$

$$\frac{17^2 - 14^2 - 25^2}{-2(14)(25)} = \cos B$$

$$\angle B \approx 40.5^\circ$$

$$\angle A \approx 32.4^\circ$$

35) $A = 50^\circ, b = 8, c = 6$

$$a^2 = 8^2 + 6^2 - 2(8)(6)\cos 50^\circ$$

$$a \approx 6.19$$

$$6^2 = a^2 + 8^2 - 2a(8)\cos C$$

$$\angle C \approx 47.967^\circ$$

$$\angle B \approx 82.033^\circ$$