Pre-Calculus

Unit 1 Review

Name Key

Name the parent function and written the transformations of each of the following.

1) $f(x) = -\frac{1}{x-2} + 11$

- Inverse or Reciprocal Function
- reflect over x-axis
- Shift right 2, up 11

2) $f(x) = \frac{1}{2}(3x+6)^3$

$f(x) = \frac{1}{2}(3(x+2))^3$

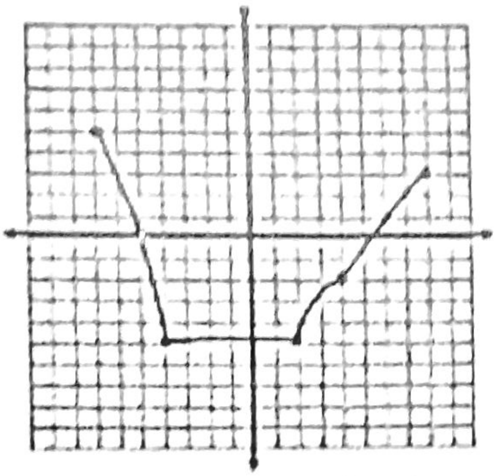
- vertical compression by $\frac{1}{2}$
- horizontal compression by $\frac{1}{3}$
- shift left 2

3) $f(x) = 2(5)^{-x+4} + 1$

$f(x) = 2(5)^{-(x-4)} + 1$

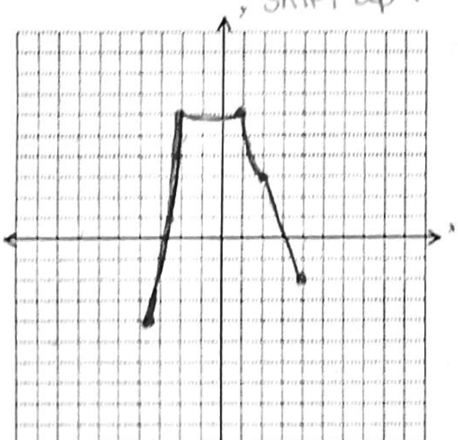
- vertical stretch by 2
- reflect over y-axis
- shift right 4, up 1

Sketch the transformation from the parent function given below.



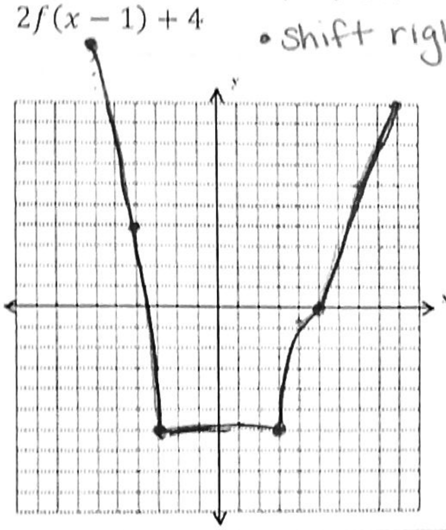
4) $-f(2x) + 1$

- reflect over x-axis
- horizontal shrink by $\frac{1}{2}$
- shift up 1



4) $2f(x-1) + 4$

- vertical stretch by 2
- shift right 1, up 4



6) Write the equation of the line that goes through the point $(6, 7)$ and is parallel to $2x + 3y = -11$

$3y = -2x - 11$

$y = -\frac{2}{3}x - 11$

$m = -\frac{2}{3}$

point-slope form $\boxed{y - 6 = -\frac{2}{3}(x - 7)}$

$y - 6 = -\frac{2}{3}x + \frac{14}{3}$

$y = -\frac{2}{3}x + \frac{14}{3} + 6$

slope-int form $\boxed{y = -\frac{2}{3}x + \frac{32}{3}}$

slope-int form $y = -\frac{2}{3}x + \frac{32}{3}$

$3y = -2x + 32$

$\boxed{2x + 3y = 32}$ Standard Form

7) Find the equation of the line that passes through the points $(3,-8)$ and $(-9,5)$. $m = \frac{5 - -8}{-9-3} = \frac{13}{-12}$

Point-slope form

$y+8 = -\frac{13}{12}(x-3)$

or

$y-5 = -\frac{13}{12}(x+9)$

Slope-int form

$y+8 = -\frac{13}{12}x + \frac{13}{4}$

$y = -\frac{13}{12}x + \frac{13}{4} - 8$

$y = -\frac{13}{12}x + \frac{13}{4} - \frac{32}{4}$

$\boxed{y = -\frac{13}{12}x - \frac{19}{4}}$

Standard Form

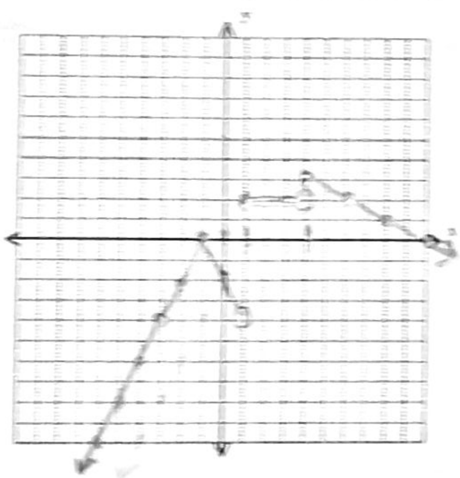
$(12)\ y = -\frac{13}{12}x - \frac{19}{4}\ (12)$

$12y = -13x - 57$

$\boxed{13x + 12y = -57}$

8) Given $f(x) = \begin{cases} -2|x+1| & x<1 \\ 2 & 1 \le x < 3 \\ 5 - \frac{1}{2}x & x \ge 3 \end{cases}$

a) Graph $f(x)$



b) State the domain.

$(-\infty, \infty)$

c) State the range.

$(-\infty, 3]$

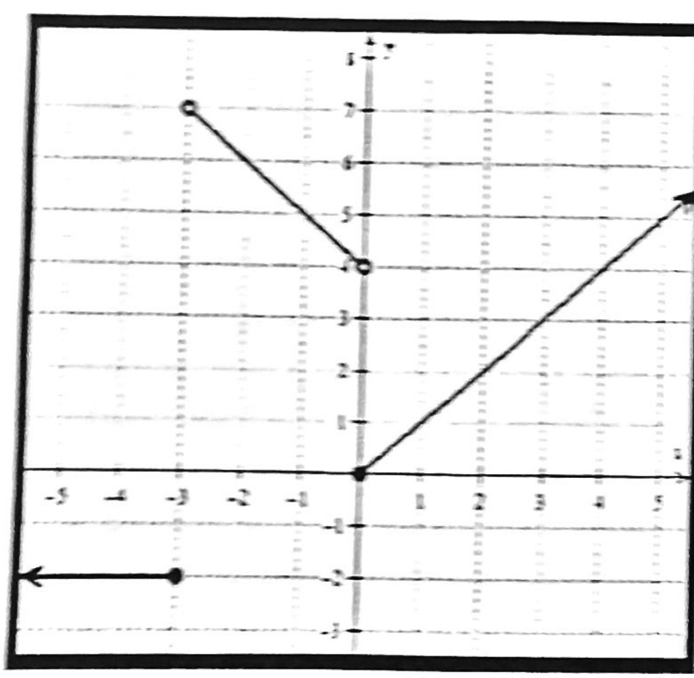
d) Find $f(-3) = -2|-3+1| = -2(2) = \boxed{-4}$

e) Find $f(3) = 5 - \frac{1}{2}(3) = 5 - 1.5 = \boxed{3.5}$

d) Find $f(1) = \boxed{2}$

---

9) Write the piecewise function for the graph below.



$f(x) = \begin{cases} -2, & x \le -3 \\ -x+4, & -3 < x < 0 \\ x, & x \ge 0 \end{cases}$

Domain: $(-\infty, \infty)$

Range: $[-2,-2] \cup [0, \infty)$

Constant: $(-\infty, -3]$

Decreasing: $(-3, 0)$

Increasing: $[0, \infty)$

Write the piecewise function to model each of the following situations.

10) An SUV was purchased for \$35,750. The value of the vehicle decreases by \$2400 a year for the first four years and \$1700 per year for the next 6 years.

$$f(t) = \begin{cases} 35750 - 2400t & 0 \le t \le 4 \\ 26150 - 1700(t-4) & 4 < t \le 10 \end{cases}$$

$$f(4) = 35750 - 2400(4) = 26150$$

11) You have a summer job that pays double time for overtime. That means if you work more than 40 hours a week, you get paid twice you hourly wage of \$8.25.

$$f(w) = \begin{cases} 8.25w, & 0 \le w \le 40 \\ 340 + 16.50(w-40), & w > 40 \end{cases}$$

$$f(40) = 8.25(40) = 340$$

12) The zoo charges \$15 per ~~hour~~ person for groups of fewer than 50 people. Groups of 50 or more are charged a reduced rate of \$10 per person.

$$f(t) = \begin{cases} 15p & , 0 \le p \le 50 \\ 10p & , p > 50 \end{cases}$$

Determine if the function is even, odd, or neither algebraically. (Use your calculator to check it graphically.)

13) $f(x) = \frac{x}{x^2+1}$

$$f(-x) = \frac{(-x)}{(-x)^2+1}$$

$$f(-x) = -\frac{x}{x^2+1}$$

$$-f(x) = -\frac{x}{x^2+1}$$

$$f(-x) = -f(x)$$

$\therefore$ odd

14) $g(x) = \sqrt{1-x^2}$

$$g(-x) = \sqrt{1-(-x)^2}$$

$$g(-x) = \sqrt{1-x^2}$$

$$g(-x) = g(x)$$

$\therefore$ Even

15) $f(x) = \frac{4\sqrt[3]{x-x^3}}{x}$

$$f(-x) = \frac{4\sqrt[3]{(-x)-(-x)^3}}{(-x)}$$

$$= \frac{4\sqrt[3]{-x+x^3}}{-x}$$

$$= \frac{4\sqrt[3]{-(x-x^3)}}{-x}$$

$$= \frac{\cancel{-}4\sqrt[3]{x-x^3}}{\cancel{-}x}$$

$$f(-x) = \frac{4\sqrt[3]{x-x^3}}{x}$$

$f(-x) = f(x)$

$\therefore$ Even

Find the domain in interval notation for each of the following functions.

16) $f(x) = \frac{2x}{x+11}$ $\neq 0$

$(-\infty, -11) \cup (-11, \infty)$

17) $f(x) = \frac{\sqrt{x+1}}{x+1}$ $x \geq -1$ $x \neq -1$

$(-\infty, -1)$

18) $f(x) = x^2 - 3x - 54$

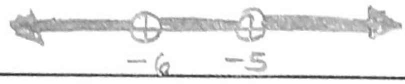
$(-\infty, \infty)$

19) $f(x) = \frac{x+2}{x^2+11x+30}$ $\neq 0$

$(x+6)(x+5) \neq 0$

$x \neq -6$ $x \neq -5$

$(-\infty, -6) \cup (-6, -5) \cup (-5, \infty)$



20) $f(x) = \frac{\sqrt{6x+1}}{x-5}$ $x \geq -1/6$ $x \neq 5$

$[-\frac{1}{6}, 5) \cup (5, \infty)$

21) $f(x) = \frac{\sqrt{x^2-1}}{x+3}$ $x \neq -3$

$x^2 - 1 \geq 0$

$x^2 \geq 1$

$x \leq -1$ or $x \geq 1$

$(-\infty, -3) \cup (-3, -1] \cup [1, \infty)$

Determine each of the following and state the domain.

$f(x) = x^2 - 4$ $(-\infty, \infty)$

$g(x) = \sqrt{x}$ $[0, \infty)$

$h(x) = \frac{3}{x}$ $(-\infty, 0) \cup (0, \infty)$

$k(x) = 2x + 3$ $(-\infty, \infty)$

22) $f(g(x)) = (\sqrt{x})^2 - 4$

$\boxed{f(g(x)) = x - 4 \quad [0, \infty)}$

23) $\boxed{(g \circ f)(x) = \sqrt{x^2 - 4}}$

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$x \leq -2$ or $x \geq 2$

$\boxed{(-\infty, -2) \cup (2, \infty)}$

24) $(h \circ g)(x) = \boxed{\frac{3}{\sqrt{x}}}$

$\boxed{(0, \infty)}$

25) $\left(\frac{k}{h}\right)(x) = \frac{2x+3}{\frac{3}{x}}$

$= \boxed{\frac{2x^2 + 3x}{3}}$

$\boxed{(-\infty, 0) \cup (0, \infty)}$

26) $\left(\frac{g}{k}\right)(x) = \boxed{\frac{\sqrt{x}}{2x+3}}$ $\boxed{[0, \infty)}$

$2x + 3 \neq 0$ $x \geq 0$

$x \neq -3/2$

27) $f(k(x))$

$(2x+3)^2 - 4$

$4x^2 + 12x + 9 - 4$

$\boxed{4x^2 + 12x + 5}$

$\boxed{(-\infty, \infty)}$

28) $(f - k)(x) =$

$(x^2 - 4) - (2x + 3)$

$x^2 - 4 - 2x - 3$

$\boxed{x^2 - 2x - 7}$ $\boxed{(-\infty, \infty)}$

29) $k(f(x))$

$2(x^2 - 4) + 3$

$2x^2 - 8 + 3$

$\boxed{2x^2 - 5}$ $\boxed{(-\infty, \infty)}$

30) $g(h(x)) = \boxed{\sqrt{\frac{3}{x}}} = \frac{\sqrt{3x}}{x}$

$= \frac{\sqrt{3}}{\sqrt{x}} = \frac{\sqrt{3} \cdot \sqrt{x}}{\sqrt{x} \cdot \sqrt{x}}$

$= \boxed{\frac{\sqrt{3x}}{x}}$ $\boxed{(0, \infty)}$

31) $f^{-1}(x) = \pm\sqrt{x+4}$ $\begin{cases} \sqrt{x+4} \\ -\sqrt{x+4} \end{cases}$

$x = y^2 - 4$

$x + 4 = y^2$

$\pm\sqrt{x+4} = y$

Inverse is not a function

$[-4, \infty)$

32) $(g \circ g)(x) = \sqrt{\sqrt{x}}$

$= \boxed{\sqrt[4]{x}}$

$\boxed{[0, \infty)}$

33) $k^{-1}(x)$ $= \boxed{\frac{x-3}{2}}$

$x = 2y + 3$

$\frac{x-3}{2} = \frac{2y}{2}$

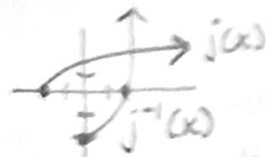
or

$\boxed{\frac{1}{2}x - \frac{3}{2}}$

$(-\infty, \infty)$

For each of the following.

a) Determine if the function is one-to-one.
b) Find the inverse of the function of the function.
c) Then state the domain and range of the function and the inverse.



24) $f(x) = \frac{x+1}{x-5}$ 1 to 1

D: $x \neq 5$ R: $y \neq 1$

$x = \frac{y+1}{y-5}$

$xy - 5x = y + 1$

$xy - y = 5x + 1$

$y(x-1) = 5x+1$

$y = \frac{5x+1}{x-1}$

$f^{-1}(x) = \frac{5x+1}{x-1}$ D: $x \neq 1$ R: $y \neq 5$

25) $g(x) = \sqrt[3]{\frac{x-2}{4}} - 5$ 1 to 1

$x = \sqrt[3]{\frac{y-2}{4}} - 5$

$(x+5)^3 = y-2$

$(x+5)^3 + 2 = y$

$g^{-1}(x) = (x+5)^3 + 2$

g(x)	g⁻¹(x)
D: $(-\infty,\infty)$	D: $(-\infty,\infty)$
R: $(-\infty,\infty)$	R: $(-\infty,\infty)$

26) $j(x) = \sqrt{x} - 2$ 1 to 1

$x = \sqrt{y} - 2$

$x + 2 = \sqrt{y}$

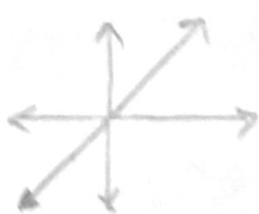
$(x+2)^2 = y$

$j^{-1}(x) = (x-2)^2$; $x \geq 0$

j(x)	j⁻¹(x)
D: $[-2,\infty)$	D: $[0,\infty)$
R: $[0,\infty)$	R: $[-2,\infty)$

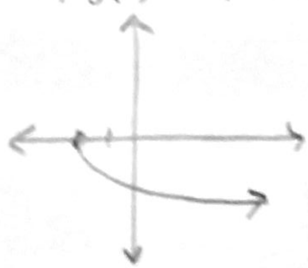
Sketch each function and State the domain and range.

27) $f(x) = x$



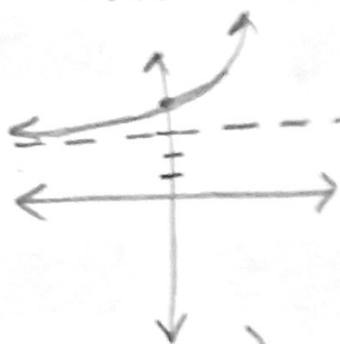
D: $(-\infty,\infty)$
R: $(-\infty,\infty)$

28) $g(x) = -\sqrt{x+2}$



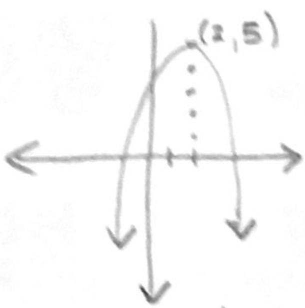
D: $[-2,\infty)$
R: $(-\infty, 0]$

29) $f(x) = e^x + 3$



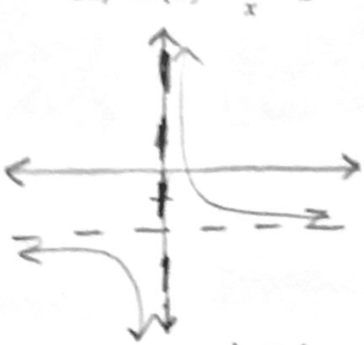
D: $(-\infty,\infty)$
R: $(3,\infty)$

30) $h(x) = (x-2)^2 + 5$



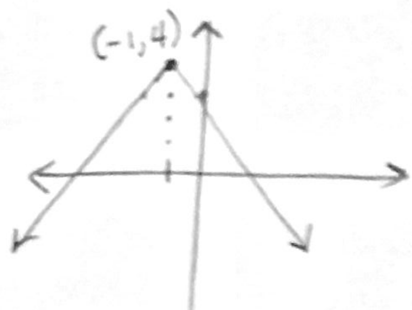
D: $(-\infty,\infty)$
R: $(-\infty, 2]$

31) $m(x) = \frac{1}{x} - 2$



D: $(-\infty,0) \cup (0,\infty)$
R: $(-\infty,-2) \cup (-2,\infty)$

32) $y = -|x+1| + 4$



D: $(-\infty,\infty)$
R: $(-\infty, 4]$