

## Converting Polar/Rectangular Equations

Convert the rectangular equation into polar form.

1)  $x^2 + y^2 - 121 = 0$

$r^2 = 121$

$r = 11$

~~$x^2 + y^2 - 49 = 0$~~

~~$r^2 = 49$~~

~~$r = 7$~~

~~$x^2 + y^2 - 28x + 22y = 0$~~

~~$r^2 - 28r\cos\theta + 22r\sin\theta = 0$~~

~~$r = 28\cos\theta - 22\sin\theta$~~

~~$x^2 + y^2 - 22x + 30y = 0$~~

~~$r^2 - 22r\cos\theta + 30r\sin\theta = 0$~~

~~$r = 22\cos\theta - 30\sin\theta$~~

5)  $x^2 + y^2 - 22x + 10y = 0$

$r^2 - 22r\cos\theta + 10r\sin\theta = 0$

$r = 22\cos\theta - 10\sin\theta$

6)  $(x-3)^2 + (y-4)^2 = 25$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 25$

$x^2 + y^2 - 6x - 8y = 0$

$r^2 - 6r\cos\theta - 8r\sin\theta = 0$

$r = 6\cos\theta + 8\sin\theta$

7)  $(x+2)^2 + (y+6)^2 = 40$

$x^2 + 4x + 4 + y^2 + 12y + 36 = 40$

$x^2 + y^2 + 4x + 12y = 0$

$r^2 + 4r\cos\theta + 12r\sin\theta = 0$

$r = -4\cos\theta - 12\sin\theta$

8)  $(x-11)^2 + (y-2)^2 = 125$

$x^2 - 22x + 121 + y^2 - 4y + 4 = 125$

$x^2 + y^2 - 22x - 4y = 0$

$r^2 - 22r\cos\theta - 4r\sin\theta = 0$

$r = 22\cos\theta + 4\sin\theta$

9)  $x = 16$

$$r \cos \theta = 16$$

$$r = \frac{16}{\cos \theta}$$

$$r = 16 \sec \theta$$

11)  $y = \frac{\sqrt{3}}{3}x$

$$\frac{y}{x} = \frac{\sqrt{3}}{3}$$

$$\theta = \frac{5\pi}{6}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$



13)  $y = 0$

$$r \sin \theta = 0$$

$$r = \frac{0}{\sin \theta}$$

$$r = 0$$

10)  $y = x$

$$\frac{y}{x} = 1$$

$$\tan \theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = \frac{\pi}{4}$$

12)  $4x + 5y = -40$

$$4r \cos \theta + 5r \sin \theta = -40$$

$$40 = -4r \cos \theta - 5r \cos \theta$$

14)  $y = 6x + 5$

$$r \sin \theta = 6r \cos \theta + 5$$

$$r \sin \theta - 6r \cos \theta = 5$$

$$5 = r \sin \theta - 6r \cos \theta$$

Convert the polar equation into rectangular form.

15)  $\theta = \frac{5\pi}{6}$

$$\tan \theta = \tan\left(\frac{5\pi}{6}\right)$$

$$\frac{y}{x} = \frac{\sqrt{3}}{3}$$

$$y = \frac{\sqrt{3}}{3}x$$

line  
slope =  $\frac{\sqrt{3}}{3}$   
y-int (0,0)

17)  $r^2 = 4^2$

$$x^2 + y^2 = 16$$

circle

center (0,0)

radius = 4

19)  $r = 4 \sin \theta$

$$r^2 = 4r \sin \theta$$

$$x^2 + y^2 = 4y$$

$$x^2 + y^2 - 4y + 4 = 0 + 4$$

$$x^2 + (y-2)^2 = 4$$

circle, center (0,2) radius = 2

16)  $\theta = -\frac{\pi}{6}$

$$\tan \theta = \tan\left(-\frac{\pi}{6}\right)$$

$$\frac{y}{x} = -\frac{\sqrt{3}}{3}$$

$$y = -\frac{\sqrt{3}}{3}x$$

line slope =  $-\frac{\sqrt{3}}{3}$   
y-int (0,0)

18)  $r^2 = 2^2$

$$r^2 = 4$$

$$x^2 + y^2 = 4$$

circle  
center (0,0) r = 2

20)  $r = -2 \sin \theta$

$$r^2 = -2r \sin \theta$$

$$x^2 + y^2 = -2y$$

$$x^2 + y^2 - 2y + 1 = 0 + 1$$

$$x^2 + (y-1)^2 = 1$$

circle center (0,1)  
r = 1

$$21) r = \frac{3}{\cos \theta}$$

$$r \cos \theta = 3$$

$$x = 3$$

Vertical line

$$23) r = 5 \cos \theta - 8 \sin \theta$$

$$r^2 = 5r \cos \theta - 8r \sin \theta$$

$$x^2 + y^2 = 5x - 8y$$

$$x^2 - 5x + \frac{25}{4} y^2 + 8y + 16 = \frac{25}{4} + 16$$

$$\left(x - \frac{5}{2}\right)^2 + (y + 4)^2 = \frac{89}{4}$$

$$\text{Center } \left(\frac{5}{2}, -4\right) \quad r = \frac{\sqrt{89}}{2}$$

$$25) r = -\frac{11}{\sin \theta}$$

$$r \sin \theta = -11$$

$$y = -11$$

horizontal line

$$27) r = 9 \sec \theta$$

$$r = \frac{9}{\cos \theta}$$

$$r \cos \theta = 9$$

$$x = 9$$

Vertical line

$$29) r = \frac{4}{1 - \cos \theta}$$

$$r - r \cos \theta = 4$$

$$r - x = 4$$

$$r = x + 4$$

$$r^2 = (x + 4)^2$$

$$x^2 + y^2 = x^2 + 8x + 16$$

$$\frac{y^2}{8} - \frac{16}{8} = \frac{8x}{8}$$

$$x = \frac{1}{8} y^2 - 2$$

parabola  
Vertex (-2, 0)

$$22) r = 8$$

$$x^2 + y^2 = 64$$

Center (0, 0)

$$r = 8$$

$$24) r = -8 \cos \theta + 10 \sin \theta$$

$$r^2 = -8r \cos \theta + 10r \sin \theta$$

$$x^2 + y^2 = -8x + 10y$$

$$x^2 + 8x + 16 + y^2 - 10y + 25 = 16 + 25$$

$$(x + 4)^2 + (y - 5)^2 = 41$$

$$\text{Center } (-4, 5) \quad r = \sqrt{41}$$

$$26) r = 4 \csc \theta$$

$$r = \frac{4}{\sin \theta}$$

$$r \sin \theta = 4$$

$$y = 4$$

horizontal line

$$28) \theta = 315^\circ$$

$$\tan \theta = \tan(315^\circ)$$

$$\frac{y}{x} = -1$$

$$y = -x$$

line  
slope = -1  
y-int (0, 0)

$$30) r = \frac{3}{3 + \sin \theta}$$

$$3r + r \sin \theta = 3$$

$$3r + y = 3$$

$$(3r)^2 = (3 - y)^2$$

$$9r^2 = 9 - 6y + y^2$$

$$9x^2 + 9y^2 = 9 - 6y + y^2$$

$$9x^2 + 8y^2 + 6y - 9 = 0$$

Ellipse

$$31) \quad r = \frac{2}{3+2\cos\theta}$$

$$3r + 2r\cos\theta = 2$$

$$3r + 2x = 2$$

$$(3r)^2 = (2-2x)^2$$

$$9r^2 = 4 - 8x + 4x^2$$

$$9x^2 + 9y^2 = 4 - 8x + 4x^2$$

$$\boxed{5x^2 + 9y^2 + 8x - 4 = 0}$$

Ellipse

$$32) \quad r = \frac{5}{2-4\sin\theta}$$

$$2r - 4r\sin\theta = 5$$

$$2r - 4y = 5$$

$$(2r)^2 = (4y+5)^2$$

$$4r^2 = 16y^2 + 40y + 25$$

$$4x^2 + 4y^2 = 16y^2 + 40y + 25$$

$$\boxed{4x^2 - 12y^2 - 40y - 25 = 0}$$

Hyperbola